

Informational frictions in financial markets

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To my loved ones.

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Erik Hapnes

Abstract

This thesis consists of three chapters on informational frictions in financial markets. The chapters analyze problems related to markets' ability to guide real investment, and what drives liquidity. Both problems are important to ensure efficient resource allocation in the economy.

The first chapter studies the interaction between financial markets and real investments. I develop a model that simultaneously study the equilibrium in financial markets, the choice of investors to produce information, and real decisions by the firm. The chapter provides a new method to overcome non-linearities in the security price, and the equilibrium is surprisingly simple. The results provide insights into when real investments have a substantial impact on market efficiency and when we can analyze equilibrium market efficiency separately. Equilibrium behavior may hide some inefficiencies from standard empirical tests. Some changes in financial markets may increase or have little effect on market efficiency, but reduce real efficiency by increasing the cost of information production.

The second chapter analyzes time-variation in liquidity. I develop a tractable model where conditions among traders vary over time. The resulting equilibrium offers several new predictions on what drives liquidity variation. For example, there may be significant reductions in liquidity from even tiny changes among the traders' conditions. Strategic behavior drives the results, and the model explains how liquidity may suddenly evaporate without a clear cause. Empirical results are in line with the predictions of the model. Surprisingly, everyone may benefit from sometimes restricting some traders from the market. Doing so can reallocate liquidity to periods with more significant liquidity needs.

The third chapter studies the choice of anonymity among traders. All traders end up revealing their identity unless doing so is costly, or the order flow is noisy. The intuition is that there is always at least one trader who prefers to reveal his or her identity. If the order flow is noisy, then there is a threshold type, and more patient traders stay anonymous. The results suggest that a fully anonymous market is most efficient, but the gains from anonymity are distributed unevenly. This result explains why different markets vary significantly in choices related to anonymity.

Zusammenfassung

Diese Arbeit besteht aus drei Kapiteln über Informationsfraktionen auf den Finanzmärkten. In den Kapiteln werden Probleme analysiert, die mit der Fähigkeit der Märkte zusammenhängen, reale Investitionen zu steuern, und was die Liquidität antreibt. Beide Probleme sind wichtig, um eine effiziente Ressourcenallokation in der Wirtschaft sicherzustellen.

Das erste Kapitel untersucht die Wechselwirkung zwischen Finanzmärkten und realen Investitionen. Ich entwickle ein Modell, das gleichzeitig das Gleichgewicht auf den Finanzmärkten, die Auswahl der Investoren für die Produktion von Informationen und reale Investitionen des Unternehmens untersucht. Das Kapitel bietet eine neue Methode zur Überwindung von Nichtlinearitäten im Wertpapierpreis, und das Gleichgewicht ist überraschend einfach. Die Ergebnisse liefern Einblicke, wann reale Investitionen einen wesentlichen Einfluss auf die Markteffizienz haben und wann wir die Effizienz des Gleichgewichtsmarktes separat analysieren können. Das Gleichgewichtsverhalten kann einige Ineffizienzen vor empirischen Tests verbergen. Einige Änderungen auf den Finanzmärkten können die Markteffizienz erhöhen oder nur geringe Auswirkungen haben, verringern jedoch die realen Effizienz, indem sie die Kosten für die Informationsproduktion erhöhen.

Das zweite Kapitel analysiert die zeitliche Variation der Liquidität. Ich entwickle ein dynamisches Modell, bei dem die Bedingungen unter den Tradern im Laufe der Zeit variieren. Das resultierende Gleichgewicht bietet mehrere neue Vorhersagen darüber, was die Liquiditätsschwankung antreibt. Zum Beispiel kann es zu einer erheblichen Verringerung der Liquidität kommen, selbst wenn sich die Bedingungen der Tradern nur wenig ändern. Strategisches Verhalten bestimmt die Ergebnisse, und das Modell erklärt, wie die Liquidität plötzlich ohne eindeutigen Grund verdunsten kann. Die empirischen Ergebnisse stimmen mit den Vorhersagen des Modells überein. Überraschenderweise kann jeder davon profitieren, manchmal einige Trader vom Markt abzuhalten. Dies kann die Liquidität auf Perioden mit höherem Liquiditätsbedarf umverteilen.

Das dritte Kapitel befasst sich mit der Wahl der Anonymität unter den Tradern. Alle Tradern geben ihre Identität preis, es sei denn, dies ist kostspielig oder es gibt viele Noise-Trader. Die Intuition ist, dass es immer mindestens einen Trader gibt, der es vorzieht, seine Identität preiszugeben. Wenn es Noise-Trader gibt, gibt es einen Schwellenwerttyp, und mehr geduldige Tradern bleiben anonym. Die Ergebnisse legen nahe, dass ein vollständig anonymer Markt am effizientesten ist, die Gewinne aus der Anonymität jedoch ungleich verteilt sind. Dieses Ergebnis erklärt, warum sich die verschiedenen

Finanzmärkte in Bezug auf die Anonymität erheblich unterscheiden.

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Introduction

Informational frictions in financial markets play an essential role in the markets' ability to function. There are informational frictions in several dimensions. For example, investors spend significant resources on analyzing firms so that they have the edge over less-informed investors. At the same time, the decision-makers in firms may also benefit from knowing the information known by informed investors. The stock price may contain information that the insiders of the firm do not know.

The interactions between informational efficiency in financial markets and the efficiency of real investments are complicated. Investors anticipate that the choices of the firm will depend on the stock price. At the same time, the firm believes that the stock price depends on the information that investors possess. Modeling this interaction is essential to understand what drives the efficiency of financial markets in equilibrium.

In the first chapter of my thesis, I develop a new method to analyze this problem. While the issue is complicated to analyze due to non-linearities, the equilibrium is surprisingly simple. The model offers several new insights. For example, if financial markets get noisier, then investors' incentives to produce information increases. This effect exactly offsets the increase in noise, and the financial market has the same ability to guide investment. There is, however, an additional cost. If more investors spend resources to produce information, then this requires resources that could have been of use for other tasks. Empirical researchers should keep such effects in mind, and changes that have a strictly negative or positive impact of real efficiency may not be observable in stock market efficiency measures.

The first chapter does not consider the trading process. Large investors' behavior directly affects the price at which they trade. To hide their full trading intentions, they optimally split their orders into multiple smaller orders. This behavior is inefficient, and there may be situations where two large investors would mutually benefit from trading, but choose only to do so gradually.

In the second chapter, I show that this behavior can have significant effects on liquidity. I analyze situations where the elements that affect traders change over time. For example, if the number of traders changes over time, there can be massive variation in liquidity that results from tiny changes in the number of active traders. The variation in liquidity appears to be an important risk factor, and the second chapter's mechanism may provide new insights into what drives this variation and how to measure it more precisely.

Most trading venues are electronic, and designing their structure can have a significant effect on their efficiency and their ability to attract customers. One dimension where different trading venues differ

significantly is in the ability to be anonymous or not. I use a game-theoretical model to analyze the behavior if investors can choose between trading anonymously or revealing their identity. Unless the market is noisy, everyone reveals their identity in equilibrium. The results of the paper also suggest that anonymity is most efficient. The two results together indicate that the efficient way to design trading venues is to make them at least partially anonymous. This structure is not necessarily the equilibrium outcome, and there may be room for welfare-enhancing policies.

1 Endogenous information acquisition with feedback effects

1.1 Abstract

I develop a theoretical model of information choice when financial markets affect real investments through learning. The model shows that if a firm can respond to information contained in security prices, then investors' incentives to gather information change. Both the specification of investment opportunities and risk affect equilibrium price efficiency. Equilibrium price efficiency may increase, decrease, or not be affected at all. Incentives to maximize the stock price may induce managers to rely solely on information from financial markets when they make real decisions. Understanding what drives the equilibrium price efficiency is, therefore, essential for sound policy advice to improve real efficiency.

1.2 Introduction

Hayek (1945) argues that prices aggregate dispersed information known to different market participants. The information contained in prices may be useful for firms' decision-makers by allowing for improved decisions and the economy by improving resource allocation. One feature of prices in financial markets is that they are forward-looking and depend on the expectations of the state of the world and firm behavior. The result is a loop where security prices affect expected firm behavior, and expected firm behavior affects security prices. This interaction is often called a feedback effect. Security prices are informative because investors spend resources to analyze the state of the world to gain an advantage in their investment decisions. The goal of this chapter is to understand how feedback effects affect investors' incentives to gather information.

The traditional view in the literature on feedback effects is that investors may have different information than the insiders of the firm in some dimensions. Bond et al. (2012) distinguish between revelatory- and forecasting price efficiency (RPE and FPE). The first is a measure of how much the insiders can learn, whereas the second measures the ability of prices to forecast returns. One may think that the insiders have superior information in most dimensions of a firm, and the ability to learn from financial markets to adjust real decisions is modest and only through the RPE. It turns out that this is not necessarily the case. If insiders' incentives are focused towards maximizing the stock price, the equilibrium behavior may be to disregard their private information and make real decisions based purely on the information that the stock price contains. The intuition is as follows: insiders

with positive information may have to overinvest to ensure that insiders with negative information do not imitate their actions. This behavior is inefficient, and a pooling equilibrium where the insiders disregard their private information may be better. The FPE may have a significant impact on real investment efficiency in a pooling equilibrium, even though the insiders learn little or no new information.

I develop a theoretical model with feedback effects and endogenous information acquisition. The results suggest that feedback effects have an ambiguous effect on price efficiency relative to the case with fixed real investment. There are situations where feedback effects have a neutral, positive, or negative effect on the equilibrium market efficiency. The magnitude of the effect can be large, with the possibility of multiple equilibria in some situations. The model nests the seminal model of Grossman and Stiglitz (1980) as a particular case, and it is straightforward to evaluate the impact of feedback effects on endogenous information acquisition.

The intuition for the ambiguity is that informed investors' informational advantage depends on the level of real investment, but not on its economic efficiency. One example is a firm that inefficiently decides to make a substantial investment. An informed investor can take a short position and benefit from the knowledge that the real investment was inefficient. The opposite example is a firm that efficiently decides not to invest in a bad project. While this is economically efficient, informed investors can not benefit from trading on their information about a project that is not undertaken. Risk also plays an essential role for informed investors. Risk-averse investors trade less aggressively on their information if they face additional risk. The exact nature of real investment behavior and risk determine the impact of feedback effects on market efficiency.

My results suggest that when a firm has an approximate symmetric ability to make modest adjustments to the size of their real investment, then we can approximate the equilibrium price efficiency well with models without feedback effects. This is no longer the case if the firm has a binary choice (e.g., invest/not invest), or the ability to make significant, symmetric adjustments to their real investments. In these cases, the equilibrium price efficiency may differ significantly from that of models without feedback effects.

The equilibrium amount of information production has significant effects on real efficiency. One example is the impact of noise trading. More noise trading increases the incentives of investors to become informed. The equilibrium price efficiency is almost independent of the amount of noise. The cost of producing information is, however, strictly increasing in the amount of noise. This result is exactly the opposite of what we would observe if information production was exogenous; more noise would reduce real investment efficiency, but not affect the information production costs. Empirical tests of models with feedback effects should carefully consider equilibrium information acquisition to account for such effects.

Solving models with feedback effects is difficult due to filtering problems and non-linear prices. Contrary to most existing models with feedback effects, I solve the model with risk-averse investors. The solution technique is slightly more complicated than in standard models of markets with asymmetric information, but the equilibrium is surprisingly tractable along several dimensions. The simplicity of the equilibrium makes the model flexible in terms of risk and real investment behavior. The tractability comes from the ability to separately analyze the security market equilibrium, the real investment choice, and the information acquisition equilibrium even though they mutually affect each other.

All solutions are either in closed form or given by a familiar fixed-point equation, and the filtering problem remains tractable.

1.3 Related literature

My model belongs to the large body of literature on noisy rational expectations equilibria (NREE) and endogenous information acquisitions (e.g., Grossman and Stiglitz (1980) Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985)). While all exogenous variables in my model are Gaussian, endogenous real investment implies that the firm payoff is non-Gaussian, making the model non-linear and potentially difficult to solve. To overcome this problem, I use insights from Breon-Drish (2015) and can solve the model explicitly.

Several papers have introduced feedback effects into NREE to study the interaction between real and informational efficiency when firms can learn from their stock price. The mechanism can work either through learning (e.g., Dow and Gorton (1997), Subrahmanyam and Titman (1999), Dow and Rahi (2003), Goldstein and Guembel (2008), Ozdenoren and Yuan (2008), Edmans et al. (2015), Sockin and Xiong (2015)) or through affecting incentives (e.g., Edmans (2009), Edmans and Manso (2011)). Models with learning and feedback effects are complicated to solve, and previous papers have used different techniques to keep the models tractable (e.g., risk neutrality, only assets in place traded, position limits). My model extends the literature on feedback effects in two dimensions, endogenous information acquisition and the introduction of risk aversion.

Dow et al. (2017) introduces endogenous information acquisition in a setting with feedback effects, risk-neutral investors, and binary investment decisions. They show that endogenous information acquisition may introduce strategic complementarities and multiple equilibria into the information market. The same effect is present in the model of this chapter for similar investment opportunities. The implications are different for other investment opportunity sets than the one with a binary choice. Both the specification of investment opportunities and risk affect the incentives to become informed. Davis and Gondhi (2019) study feedback effects and endogenous information acquisition. They focus on the impact of endogenous information and conflicts between shareholders and creditors.

The models by Dow and Rahi (2003), Bond and Goldstein (2015), and Siemroth (2019) are, to my knowledge, the only other models with learning, feedback effects, and risk-averse investors. Bond and Goldstein (2015) and Siemroth (2019) study the effect of government intervention when the stock price contains useful information. Real investment and government interventions have one important difference in how they affect the security market equilibrium. The main difference can be illustrated with a firm that has a payoff of the following form:

$$V = I(s + \varepsilon) - C(I) + T$$

Here, $s + \varepsilon$ is the state of the world, I is the real investment, $C(I)$ is adjustment costs, and T is the government intervention. In this chapter, I focus on variation in I , which enters multiplicatively with $s + \varepsilon$. This causes the risk premium to have complicated non-linearities. Bond and Goldstein (2015) and Siemroth (2019) focus on T , which requires different methods to account for feedback effects. Dow and Rahi (2003) also studies real investment, but they clear the market with a risk-neutral market maker, which removes the risk premium. Their model is a special case of the security market I present

in this chapter.

Empirical studies have shown that real investment is sensitive to stock price movements (e.g., Barro (1990), Morck et al. (1990)). Managers learning decision-relevant information from their stock price is one mechanism that explains these findings. Several studies have found support for this mechanism (e.g., Luo (2005), Chen et al. (2007), Bakke and Whited (2010), Foucault and Fresard (2014), David et al. (2016), Bai et al. (2016), Edmans et al. (2017), and Dessaint et al. (2019)). A problem for empirical research is that it is difficult to disentangle the effect of learning from other mechanisms that make real investment sensitive to stock price movements. I provide new empirical predictions that depend on the endogenous information acquisition, some of which differ significantly from the predictions with exogenous information acquisition.

1.4 Model setup

Figure 1.1 – Timeline



This section presents the baseline model for investment and information acquisition with feedback effects. See Appendix A.1 for extensions with dispersed information, and more general specifications of risk and investment behavior.

Agents. The model has four kinds of agents: informed investors, uninformed investors, noise traders, and the firm manager.

- *Informed investors.* Rational investors who observe a signal about the final payoff.
- *Uninformed investors.* Rational investors who learn about the final payoff from observing the stock price.
- *Noise traders.* Traders who trade for reasons unrelated to the final payoff.
- *Firm manager.* The firm manager chooses the investment level after observing the stock price.

Securities. There are two assets: one risk-free asset with interest rate normalized to 0 and one risky asset. The risky asset has a final payoff V given by

$$V = I(s + \varepsilon) + \varepsilon_0 - \frac{c}{2}(I - I_0)^2.$$

The payoff is a function of real investment I and three random parts, s , ε , and ε_0 . Some assets that are unrelated to the real investment behavior with payoff ε_0 can be in place. The random parts are independent and normally distributed with mean and variance given by $s \sim N(0, \sigma_s^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, and $\varepsilon_0 \sim N(\bar{\varepsilon}_0, \sigma_{\varepsilon_0}^2)$. The asset supply for informed and uninformed investors is $\bar{z} - z$ where z is the demand of the noise traders. Noise traders demand $z = \frac{I}{I^2 + \frac{\sigma_0^2}{\sigma_\varepsilon^2}} \tilde{z} \equiv \hat{f}(I) \tilde{z}$ where $\tilde{z} \sim N(0, \sigma_z^2)$. This functional

form is crucial for the solution technique¹. The equilibrium signal-to-noise ratio is independent of realizations of the random variables with this specification of $\hat{f}(I)$, which simplifies the filtering problem. The market clearing condition is

$$\lambda AX_i + (1 - \lambda)AX_u = \bar{z} - z$$

where X_i and X_u are the demands of informed and uninformed investors. Here, A is a parameter that I name "the size of the market," equal to the total mass of rational investors. This parameter is redundant in the security market, but it simplifies the analysis of the information market. A fraction λ of the investors are informed and $(1 - \lambda)$ are uninformed.

Information structure. Informed investors pay a cost k to observe s . The uninformed investors update their beliefs after observing the security price. The manager of the firm may have information about the final payoff but is not able to reveal this information truthfully to investors. I do not put any restrictions on the information set of the manager.

Trading rounds. There are two trading rounds, one before and one after the real investment choice. There is no new information between the two trading rounds other than what is revealed by the real investment choice. I assume that a random fraction, $\beta \in (0, 1)$, of the investors can only trade in round 1. This assumption does not affect anything in equilibrium but ensures that the optimization problem is well-defined in both rounds. The price in the first trading round is P , whereas the price in the second trading round is P_2 .

Real investment opportunities. To ensure that $\hat{f}(I)$ is finite, I require that either $I > 0$ or $\sigma_0^2 > 0$. I do not put other restrictions on the investment opportunity set, \mathcal{I} . In equilibrium, the choice of the investment opportunity set has important implications for the equilibrium behavior of investors when they make their decisions about becoming informed or not.

Preferences. Informed and uninformed investors maximize utility with constant absolute risk aversion a and act as price takers. The demanded quantity is

$$X_j = \arg \max_{X_j} E[-e^{-aW_j} | \mathcal{F}_j]$$

where $W_j = X_j(V - P)$ and $j = i$ for informed investors and $j = u$ for uninformed investors. The manager is risk-neutral and chooses the investment level to maximize the stock price²,

$$I = \arg \max_{I \in \mathcal{I}} E[P_2 | P, \mathcal{F}_m].$$

¹A simple method to endogenize the noise traders is to include some irrational traders who observe \bar{z} , and behave as if they observed s . Banerjee and Green (2015) use this technique to obtain closed form solutions in a model with non-linearities. An alternative is to include idiosyncratic risk that is correlated with s .

²The solution technique allows the manager to have other preferences. This requires additional assumptions on the parameters of the model and the manager's information set. Conditional on this being the case, the condition in the information market equilibrium is not affected, and the implications for the equilibrium price efficiency are similar.

1.5 Model solution

As is common in the literature, I consider equilibria where the equilibrium price is increasing in the signal observed by uninformed investors. The equilibrium price is non-linear, and the standard solution methods are inappropriate. I solve the model with the method first developed by Breon-Drish (2015). The first step is to find the optimal demand for informed investors. The optimal demand is combined with the market clearing condition to find the signal that can be extracted by the uninformed investors. The last step is to find the uninformed investors' demand given their beliefs and the price such that the market clears. I abuse notation and write I instead of $I(P)$ throughout this section for readability.

Definition 1. A Perfect Bayesian Equilibrium consists of demand functions X_i and X_u for informed and uninformed traders in both trading rounds, price functions P and P_2 , and beliefs \mathcal{F}_j for all agents j in both trading rounds such that

- (i) X_j is optimal for type j conditional on \mathcal{F}_j in both trading rounds.
- (ii) The asset markets clear for all s and \tilde{z} in both trading rounds.
- (iii) \mathcal{F}_j satisfy Bayes's rule on the equilibrium path.

This section only deals with the existence of an equilibrium and not its uniqueness for two reasons. Pálvölgyi and Venter (2015) show that there are multiple equilibria in the model by Grossman and Stiglitz (1980) when they allow for discontinuous price functions. Such equilibria are likely to be present in the model of this chapter for the same reasons. Furthermore, I look for equilibria where the real investment is only a function of P . Solving the model without this assumption is complicated as the final payoff is no longer conditionally normal. While this may rule out some interesting equilibria, it is nevertheless surprising that such an equilibrium always exists regardless of the information set of the manager. The following Lemma is useful for solving the model:

Lemma 1. If $P_2 = P$ a.s., then all investors behave as if they will be restricted from trading in the second round when they make their portfolio decisions in the first round.

Proof: See appendix A.2.1.

The Lemma is useful because we can analyze a simpler model where traders only make their decisions once on the equilibrium path. It still allows for $P_2 \neq P$ out-of-equilibrium, which is required to ensure that the optimization problem of the manager is well-defined.

1.5.1 Informed demand

Informed investors use demand schedules and can condition their demand on s and the price. They solve

$$\max_{X_i} E[-e^{-aW_i} | s, P]$$

where $W_i = X_i(V - P)$. Lemma 2 gives the solution to the maximization problem.

Lemma 2. *The demand of informed investors is*

$$X_i = \frac{E[V|s, P] - P}{a \text{Var}[V|s, P]} = \hat{f}(I) \frac{s}{a\sigma_e^2} - \hat{f}(I) \frac{P + \frac{c}{2}(I - I_0)^2 - \bar{s}_0}{aI\sigma_e^2}.$$

Proof: See appendix A.2.2.

The first term is linear in $s\hat{f}(I)$, and this is the crucial element that keeps the security market equilibrium tractable. The second term is a function of P , and uninformed investors can infer its value.

1.5.2 Market clearing and residual supply

Combine the market clearing condition with the demand of informed investors to obtain

$$\lambda A \left(\hat{f}(I) \frac{s}{a\sigma_e^2} - \hat{f}(I) \frac{P + \frac{c}{2}(I - I_0)^2 - \bar{s}_0}{aI\sigma_e^2} \right) + (1 - \lambda) AX_u(P) = \bar{z} - z.$$

Residual supply for uninformed investors is

$$(1 - \lambda) AX_u(P) = \bar{z} - \frac{\lambda A}{a\sigma_e^2} \hat{f}(I) \left(s + \frac{a\sigma_e^2}{\lambda A} \bar{z} \right) + \lambda A \hat{f}(I) \frac{P + \frac{c}{2}(I - I_0)^2 - \bar{s}_0}{aI\sigma_e^2}.$$

Define $s_u = s + \frac{a\sigma_e^2}{\lambda A} \bar{z}$. This is the signal that uninformed investors and the manager can extract in equilibrium.

Assumption 1. *The equilibrium price is strictly increasing in s_u for realizations where $I(s_u) > 0$ and increasing otherwise.*

Assumption 1 is similar to the standard approach of guessing a linear price and verifying that it is an equilibrium. Feedback effects make the price non-linear and we have to weaken the assumption to an increasing rather than linear price. Both uninformed investors and the manager can extract s_u after observing the stock price. We can write both uninformed demand, the price P and real investment as functions of s_u . This simplifies the market clearing condition to

$$(1 - \lambda) AX_u(s_u) = \bar{z} - \frac{\lambda A}{a\sigma_e^2} \hat{f}(I) s_u + \lambda A \hat{f}(I) \frac{P + \frac{c}{2}(I - I_0)^2 - \bar{s}_0}{aI\sigma_e^2}.$$

1.5.3 Uninformed demand

Uninformed investors have the same maximization problem as informed investors, but they can only condition expected utility on s_u . Their maximization problem is

$$\max_{X_u} E[-e^{-aW_u} | s_u].$$

Uninformed investors update their beliefs about V after observing s_u . Define

$$c_0 = \frac{a\sigma_e^2}{\lambda A}.$$

Conditional on observing the security price, the value of V has a normal distribution with

$$E[V|s_u] = Ic_1 s_u - \frac{c}{2}(I - I_0)^2 + \bar{s}_0$$

and

$$\text{Var}[V|s_u] = I^2(1 - c_1)\sigma_s^2 + I^2\sigma_e^2 + \sigma_0^2$$

where

$$c_1 = \frac{\sigma_s^2}{\sigma_s^2 + c_0^2\sigma_z^2}. \quad (1.1)$$

The constant c_1 takes a value between 0 and 1 where the price is uninformative if $c_1 = 0$ and fully reveals s when $c_1 = 1$. I use c_1 as the measure of price efficiency throughout the chapter.

Lemma 3. *The demand of the uninformed investors is*

$$X_u = \frac{E[V|s_u] - P}{a\text{Var}[V|s_u]} = \frac{Ic_1 s_u - \frac{c}{2}(I - I_0)^2 - P + \bar{s}_0}{a(I^2(1 - c_1)\sigma_s^2 + I^2\sigma_e^2 + \sigma_0^2)}.$$

Proof: See appendix A.2.3.

1.5.4 Equilibrium in the security market

We need to find a price such that the market clears given the demand of the uninformed investors. Define a_i as $\frac{a}{\lambda A}$ and a_u as $\frac{a}{(1-\lambda)A}$ to simplify notation. The market-clearing condition is given by

$$\hat{f}(I)\left(\frac{s_u}{a_i\sigma_e^2} - \frac{P + \frac{c}{2}(I - I_0)^2 - \bar{s}_0}{a_i I\sigma_e^2}\right) + \frac{Ic_1 s_u - (P + \frac{c}{2}(I - I_0)^2 - \bar{s}_0)}{a_u(I^2(1 - c_1)\sigma_s^2 + I^2\sigma_e^2 + \sigma_0^2)} = \bar{z}. \quad (1.2)$$

To find an equilibrium, we need to solve (1.2) for P , find the optimal investment I , and verify that the price function is increasing in s_u .

Proposition 1. *The unique solution to equation (1.2) is*

$$P(s_u; I) = \underbrace{\bar{s}_0 + E[s|s_u]I - \frac{c}{2}(I - I_0)^2}_{\text{Expected value}} - \underbrace{\frac{d_1(I)d_2(I)}{d_1(I) + d_2(I)}\bar{z} + (1 - c_1)\frac{d_1(I)}{d_1(I) + d_2(I)}I s_u}_{\text{Risk premium}} \quad (1.3)$$

where

$$d_1(I) = a_u(I^2((1 - c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2)$$

$$d_2(I) = a_i(I^2\sigma_e^2 + \sigma_0^2).$$

There is an equilibrium where the optimal real investment level,

$$I^*(s_u) = \arg \max_I P_2(s_u; I),$$

is a function of s_u and the price, $P(s_u; I^*(s_u))$, is increasing in s_u . $P(s_u; I^*(s_u))$ is strictly increasing when $I^*(s_u) > 0$.

Proof: See appendix A.2.4.

The equilibrium price can be split into two elements, the expected payoff, and the risk premium. The non-linearities in the risk premium are non-trivial. These non-linearities are the main difficulty of solving models with feedback effects in the standard noisy rational expectations models. One special case is $A \rightarrow \infty$ where $a_u \rightarrow 0$. In this case, the risk premium disappears, and $I(s)$ is a linear function of s_u . This special case is analyzed by Dow and Rahi (2003). $I^*(s_u)$ is linear if $A \rightarrow \infty$ or $\sigma_0^2 = 0$, and non-linear otherwise.

The precision of the signal that the uninformed investors and the manager can observe is given by c_1 . This is a constant for fixed values of λ , and the learning in the security market equilibrium is solely a filtering problem. To understand how the feedback between secondary financial markets and real investment affects price efficiency, we need to endogenize λ .

1.5.5 Equilibrium in the information market

Three possible types of equilibria can be found in the information market: (1) All traders strictly prefer to be uninformed, (2) all traders strictly prefer to be informed, or (3) all traders are indifferent between being informed or uninformed. Define $\Delta_U(c_1)$ as the unconditional difference in the expected utility of the informed and uninformed investors.

Definition 2. *The equilibrium price efficiency satisfies $\Delta_U(c_1) = 0$, $\Delta_U(0) < 0$ or $\Delta_U(1) > 0$.*

If $\Delta_U(c_1) = 0$, there exists an interior solution where a fraction $\lambda \in (0, 1)$ of the investors is informed. If $\Delta_U(0) < 0$, an equilibrium exists where no investors are informed and if $\Delta_U(1) > 0$ an equilibrium exists where all investors are informed.

Proposition 2. *The difference between the expected utility of an informed investor and an uninformed investor is*

$$\Delta_U(c_1) = E \left[\left(1 - e^{ak} \sqrt{\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}} \right) e^{-\frac{(E[V|s_u] - P(s_u))^2}{2\text{Var}(V|s_u)}} \right].$$

Proof: See appendix A.2.5.

The equilibrium condition for an equilibrium with $\lambda \in (0, 1)$ in the information market is similar to the condition in Grossman and Stiglitz (1980). $1 - e^{ak} \sqrt{\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}}$ is a constant when investment is fixed and the rest is an expectation of a positive random variable. The ability to adjust investment usually changes this condition and we have to evaluate an expectation. The equilibrium condition simplifies for a large market ($A \rightarrow \infty$).

Corollary 1. *When the market is large ($A \rightarrow \infty$), then:*

$$\lim_{A \rightarrow \infty} \Delta_U(c_1) = E \left[\left(1 - e^{ak} \sqrt{\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}} \right) \right].$$

Proof. Follows from the bounded convergence theorem and $\lim_{A \rightarrow \infty} \frac{(E[V|s_u] - P(s_u))^2}{2\text{Var}(V|s_u)} = 0$. □

Corollary 1 simplifies the information equilibrium condition and makes it easier to obtain theoretical results. Numerical results are similar except when $\lambda \approx 1$ where a_u is large. For notational simplicity, define c_1 and \bar{c}_1 as the equilibrium price efficiency with and without feedback effects.

1.6 Equilibrium information acquisition

The equilibrium implications of feedback effects rely crucially on investment opportunities. In this section I compare the incentives to become informed with and without feedback effects for different types of real investments. To be precise, the setting without feedback effects is equivalent to $c \rightarrow \infty$ such that $I(s_u) = I_0$ if $I_0 \in \mathcal{I}$. The following Lemma is useful to analyze the information equilibrium:

Lemma 4. *Define $g(I) = 1 - e^{ak} \sqrt{\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}}$.*

1. *If $\sigma_0^2 = 0$, then:*

- *$g(I)$ is constant.*

2. *If $\sigma_0^2 > 0$, then:*

- *$g(I)$ is increasing.*
- *$g(I)$ is strictly concave for $I < \bar{I}$ and strictly convex for $I > \bar{I}$ where \bar{I} is the unique positive solution to (A.5).*

Proof: see appendix A.2.6.

Each of the following examples shows a case where one of the elements of Lemma 4 is important. All three cases are used in the existing feedback literature, and my results suggest that they have different impacts on the incentives to become informed.

1.6.1 No unrelated assets

The simplest situation to analyze is one without risky assets that are unrelated to I (i.e. $\sigma_0^2 = 0$). It is not necessary to add additional assumption on any parameters to analyze this situation (other than $I > 0$).

Corollary 2. Let $\mathcal{I}_0 = \{I|I > 0\}$, $\mathcal{I} \subseteq \mathcal{I}_0$, and $\sigma_0^2 = 0$. Then there is a unique equilibrium in the information market that is independent of real investment behavior. The equilibrium price efficiency in an equilibrium with $\lambda \in (0, 1)$ is

$$c_1 = 1 - \frac{\sigma_e^2}{\sigma_s^2} (e^{2ak} - 1).$$

Proof: see appendix A.2.7.

The result of Corollary 2 suggests that analyzing the equilibrium in the information market separately from the interaction between financial markets and real investment decisions can be a good approximation. The reason is that the informational advantage of informed investors depends on $\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}$, which is constant when $\sigma_0^2 = 0$.

1.6.2 Binary investment

Dow et al. (2017) analyze endogenous information production with feedback effects for firms with a binary choice, $I \in \{I_1, I_2\}$.

Corollary 3. Let $\mathcal{I} = \{I_1, I_2\}$ with $I_2 > I_1$.

(1) If $I_0 = I_2$, then:

- $0 < c_1 \leq \bar{c}_1$ if $k < \bar{k}_2 = \frac{1}{2a} \log \left(\frac{I_2^2(\sigma_s^2 + \sigma_e^2) + \sigma_0^2}{I_2^2\sigma_e^2 + \sigma_0^2} \right)$. $c_1 = \bar{c}_1 = 0$ otherwise. Furthermore, $c_1 < \bar{c}_1$ if $\lambda < 1$.

(2) If $I_0 = I_1$, then:

- $\bar{c}_1 = 0$ if $k \geq \bar{k}_1 = \frac{1}{2a} \log \left(\frac{I_1^2(\sigma_s^2 + \sigma_e^2) + \sigma_0^2}{I_1^2\sigma_e^2 + \sigma_0^2} \right)$. $\bar{c}_1 \leq c_1$ with $\bar{c}_1 < c_1$ if $\bar{c}_1 > 0$ and $\lambda < 1$.
- $c_1 = 0$ is an equilibrium if $k \geq \bar{k}_1$. If $I_0 = 0$ and k is sufficiently small, then there are at least three equilibria, $c_1 = 0$, and two equilibria with $c_1 > 0$.

Proof: see appendix A.2.8.

The real investment opportunities and equilibrium implications are similar in Corollary 2 and the model by Dow et al. (2017). Feedback effects can have large implications for the incentives to produce information. If the precision of s_u increases, then the probability of $I(s_u) = I_2$ increases (decreases), which increases (decreases) the incentives to become informed.

The mechanism behind the result is that informed investors prefer I to be larger so that they can benefit more from their information. As a result, the incentives to produce information increases if the firm gets an option to invest. The opposite is the case if they get an option to reduce the scale of their operations.

One example is a firm that can choose different directions for an R&D project, which differ in the size of the final investment (e.g., $\mathcal{I} = \{0, I_2\}$ for different values of I_2). The optimal project choice depends

on both the equilibrium price efficiency and the size of the project. A larger project may be better for a fixed value of c_1 , but may not support an equilibrium with $c_1 > 0$. The reason is that the probability that the firm will undertake a large project is lower, which reduces the incentives to become informed.

1.6.3 Symmetric investment

Another frequently used setting is one where the firm can symmetrically adjust investment up or down after good or bad information (e.g., Edmans et al. (2015)). The exact specification here is chosen so that the concavity or convexity of $g(I)$ drives the results.

Corollary 4. *Let $\mathcal{I} = \{I_0(1 - \epsilon), I_0, I_0(1 + \epsilon)\}$ with $\epsilon \in (0, 1)$, $A \rightarrow \infty$, and the parameters are such that $\bar{c}_1 > 0$.*

(1) *If $\Delta(\bar{c}_1) \gtrless 0$, then there exist a $c_1 \gtrless \bar{c}_1$ where $\Delta(c_1) = 0$.*

(2) *$\Delta(\bar{c}_1)$ can be approximated by*

$$\Delta(\bar{c}_1) = C \left((3 - 2e^{2ak})\sigma_0^2 - 3e^{2ak}I_0^2\sigma_e^2 \right) \epsilon^2 + O(\epsilon^4)$$

where C is a positive constant.

Proof: see appendix A.2.9.

Corollary 4 suggests that the ability to adjust the real investment symmetrically has an ambiguous effect on the equilibrium price efficiency. When σ_0^2 is small, then the equilibrium price efficiency would drop and vice versa if σ_0^2 is large.

The numerical example in Figure 1.2 illustrates that the effect can be significant, especially when ϵ is large. The example is constructed with parameters such that $\bar{c}_1 = \frac{1}{2}$. The difference between c_1 and \bar{c}_1 is tiny for $\epsilon = 0.1$ and has the signs we would expect from Corollary 4. This is no longer the case when $\epsilon = 0.9$. In this situation, there may be a large change in price efficiency. The effect is largest when σ_0^2 is positive but small. This result is surprising in light of Corollary 2. The equilibrium price efficiency is independent of real investment behavior when $\sigma_0^2 = 0$, and we would expect that this is a good approximation when σ_0^2 is small. The numerical example in Figure 1.2 shows that this is not always the case.

1.7 Real efficiency

The real efficiency in the model depends on two elements, the efficiency of real investments and the information production costs, defined as:

$$W(c_1) = \underbrace{E[V]}_{\text{Expected payoff}} - \underbrace{\lambda Ak}_{\text{Cost of information production}}$$

Understanding what drives real efficiency is crucial to implement sound policies. Furthermore, we need to know how to measure the impact of different policies to evaluate their performance. Both

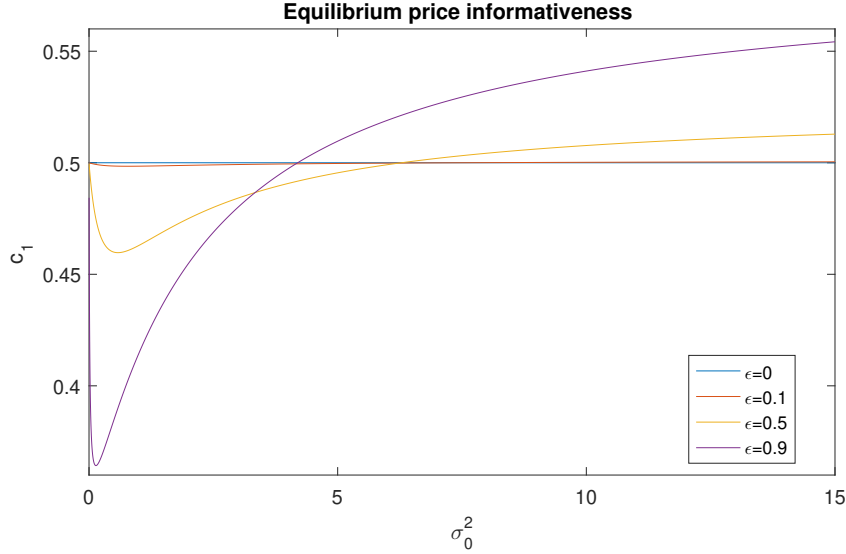


Figure 1.2 – The plot shows the equilibrium price efficiency for different values of ϵ and σ_0^2 . The remaining parameters are as follows: $a = 1$, $k = \frac{\log(5) - \log(4)}{2}$, $\sigma_e^2 = 1$, $\sigma_s^2 = \frac{\sigma_0^2 + 1}{2}$, $I_0 = 1$, and the value of c ensures that $P(I = I_0 - \epsilon) = P(I = I_0 + \epsilon) = \phi(-\frac{1}{4})$ where $\phi(\cdot)$ is the normal CDF if $c_1 = \frac{1}{2}$. These parameters are chosen to have $\bar{c}_1 = \frac{1}{2}$.

elements are complicated when information production is endogenous. Policies can potentially have an impact on at least two of the parameters of the model. Regulations can have an impact on the amount of noise in the financial markets, for example, by regulating how easy it is for retail investors to trade directly or whether their identity should be revealed. Another dimension is the cost of information production, k . It may be possible to influence the cost in either direction, either by the firm itself or by the government.

Proposition 3. *Let $A \rightarrow \infty$. Then:*

- (1) *The expected value of the final payoff, $E[V]$, is strictly increasing in c_1 .*
- (2) *There exists a $\bar{\sigma}_z^2$ such that $W(c_1) \geq W(0)$ if $\bar{\sigma}_z^2 \geq \sigma_z^2$.*
- (3) *A change in σ_z^2 has a neutral effect on the equilibrium price efficiency ($\frac{\partial c_1}{\partial \sigma_z^2} = 0$). An increase in σ_z^2 strictly reduces $W(c_1)$.*
- (4) *An increase in the cost of information, k , reduces equilibrium price efficiency ($\frac{\partial c_1}{\partial k} < 0$), but has an ambiguous effect on $W(c_1)$. An increase in k increases $W(c_1)$ if $\frac{\partial c_1}{\partial k} < -2c_1(1 - c_1)$ and σ_z^2 is sufficiently large.*

Proof: see appendix A.2.10.

Proposition 3 shows how different elements affect real efficiency. The first part is not surprising; a more informative price increases the real investment efficiency. Whether this increase in investment efficiency is greater than the cost of producing information is, however, ambiguous. Investors do not produce information to increase the efficiency of real investments, but rather to gain a benefit when

they trade against less informed traders. If there is more noise in the market, then it is more costly to produce information relative to the improvements in real investment decisions.

Changes in the amount of noise has a neutral effect on the equilibrium market efficiency. More noise increases the incentives to produce information. More information production exactly offsets the effect of noise on market efficiency. As a result, the efficiency of real investment is not affected by the amount of noise trading. The real efficiency is, however, affected by the cost of information production. The empirical prediction is that noise trading may have a small effect on real investment efficiency, but a more significant effect on the number of investors who analyze firms. If information production was exogenous, then the empirical prediction would be the exact opposite.

A reduction in the cost of becoming informed, k , increases the equilibrium price efficiency and real investment efficiency. The total effect on the cost of information production is ambiguous. There are situations where the total cost of information production increases. An empiricist would expect to find that a reduction in k increases the efficiency of real investment and the number of informed investors. This empirical prediction differs from the partial equilibrium with exogenous information production, where the only change would be a reduction of costs for informed investors.

1.8 Conclusion

I show that it may be optimal for managers to put a high weight on information from financial markets when they make their decisions. Therefore, it is crucial to understand what drives the equilibrium price efficiency, both for firms and regulators. One example is the impact of noise traders. They have little impact on the price efficiency in equilibrium, but they do affect the cost of information production. This is important for empirical studies that try to understand their effects on real efficiency.

The main thing that affects the equilibrium price efficiency in this chapter is the real investment behavior. The results suggest that we can approximate it well with results without feedback effects (e.g., Grossman and Stiglitz (1980)) when the ability to adjust the size of real investment is not too large. This changes when the firm can make significant changes to their real investments. Firms can potentially use these results to influence the efficiency of their security prices, which again will help them to make better decisions.

The focus of this chapter was on the equilibrium information acquisition. The other main contribution of this chapter is to introduce feedback effects in a model with a risk premium that depends on real investment. This property makes it a natural starting point to study asset pricing implications of feedback effects. One example is that a higher price increases real investment, which again raises the risk premium; the empirical implication of this mechanism is time-series momentum. It is purely risk-based, and we do not require any behavioral explanation with under-/overreaction.

2 Time-varying liquidity with strategic traders

2.1 Abstract

I investigate how shocks to large, strategic traders produce time-variation in liquidity. I develop a dynamic model in which competition and the cost of holding inventories change over time. Anticipating future shocks to market composition, strategic traders optimally choose when to trade, affecting the dynamics of liquidity. Even small shocks to the degree of competition can cause significant drops in liquidity if these shocks are short-lived. The model offers guidance on how to measure the impact of liquidity shocks of different types and provides new insights on the welfare effects of time-varying liquidity.

2.2 Introduction

A well-functioning financial market provides immediacy for investors. For example, an investor who wants to sell a significant position can do so at a price close to the current bid if the market is liquid. The ability of the market to provide immediacy varies over time. Even the most liquid financial markets may suffer from sudden evaporation of liquidity that is entirely unrelated to the underlying asset fundamentals. Many of such episodes originate from a shock to a single, large, strategic trader. How can it be that a small change in the composition of key traders has such a large impact on market liquidity? Why do other large traders often abstain from replacing the distressed trader and refuse to provide liquidity to each other? The goal of this chapter is to answer these questions in a model of dynamic, strategic trading.

The two most common explanations for illiquidity in financial markets are adverse selection and inventory-absorption capacity. I follow the latter approach. In my model, all traders are strategic, and their costs of holding inventory and the ability to trade change over time. I show that combining this time-variation with strategic behavior adds several novel insights. The reason is that strategic traders choose both how much to trade and when to do so. They optimally decide to trade more in high liquidity periods and less in low liquidity periods, amplifying endogenous liquidity fluctuations. The resulting feedback loop implies that even small shocks to the composition of traders can have a significant effect on liquidity. Furthermore, I show that different types of liquidity shocks affect liquidity measures differently.

The model has two price impacts, a temporary and a permanent price impact. Short-lived shocks to competition have a significant effect on the temporary price impact, and my model suggests that they will dominate empirical measures of this price impact. Short-lived shocks have a small effect on the permanent price impact. The model suggests that persistent liquidity shocks mostly drive variation in the permanent price impact. For example, my model provides a rationale for the low correlations between the two measures of liquidity by Pástor and Stambaugh (2003) and Nagel (2012). The measure of Pástor and Stambaugh (2003) estimates the temporary effect on returns from volume. The theoretical counterpart in my model is the temporary price impact. Nagel (2012) uses return reversal strategies as a measure of the market's ability to absorb inventory imbalances. The theoretical counterpart in my model is the permanent price impact. My theoretical model suggests that the first measure should be dominated by large, negative, and short-lived liquidity shocks, whereas the second measure has smaller, but more persistent shocks. Both of the empirical measures show patterns that are in line with the predictions of my model.

One example that illustrates several key aspects of my model is Knight Capital, a former market maker. They suffered huge losses due to a trading glitch originating from an error in their electronic systems. Liquidity deteriorated both in stocks affected by the trading glitch and unaffected stocks where they served as the designated market maker (see Bogousslavsky et al. (2018)). Other traders replaced Knight Capital as designated market makers within days, and liquidity improved. The trading glitch initially affected prices, but the effect on prices reverted much quicker than liquidity. This event illustrates several insights from the model in this chapter. Even small shocks to the composition of market participants can have a significant effect on liquidity if they are short-lived. The initial price effect can be explained by out-of-equilibrium behavior during the trading glitch. Current trading behavior affects the beliefs about future trading intentions, and the trading glitch could have had a large initial effect on prices through these beliefs. When other traders understood that the behavior was due to an error and not due to large trading needs, prices should quickly reverse.

My results suggest that large shocks to liquidity are more prevalent in markets with few participants. This finding is important for policies that aim to improve the stability of financial markets. One such policy is the "Volcker Rule," which prohibits banks from proprietary trading. The rule allows for banks to act as market makers, but the distinction between the two activities is not always clear (see Duffie (2012) for a detailed discussion). Regulations such as the "Volcker Rule" may reduce the number of market participants who provide liquidity and increase the market power for those who remain active. A more concentrated market increases the likelihood of large liquidity shocks and may introduce new, systematic risks in liquidity.

A recent concern in financial markets is flash crashes, which have occurred in several large and highly liquid markets. Common features with these crashes is that they are extremely short-lived, the effect on prices is large, and they have no clear event that caused them. My model can explain how liquidity evaporates in these short-lived shocks. If some traders still trade during these periods, then they have a large impact on prices. One notable event is the flash crash of May 6, 2010. Evidence from Easley et al. (2011) and Kirilenko et al. (2017) suggests that high-frequency traders behavior can contribute to such events. CFTC and SEC (2010) report that several market participants stopped trading around the crash. Breedon et al. (2019) study trading behavior around the Swiss franc cap removal. They find that algorithmic traders withdrew liquidity, whereas human trades did the opposite. Evidence from both events suggests that algorithmic traders might stop to provide liquidity around large shocks,

potentially exacerbating the magnitude of liquidity shocks.

The duration of a liquidity shock has different implications for different market participants. A highly levered investor, one prominent example is Long Term Capital Management, may be forced to liquidate their positions exactly when it is costly to do so. Even short periods with low liquidity can induce sufficiently large losses to put them into bankruptcy. Other traders are more patient and can choose not to trade during low liquidity periods. One example is an activist investor who gradually acquires a significant position. He or she can choose when to trade and reduce the total transaction costs.

2.3 Related literature

The chapter builds on the large literature on strategic trading (e.g., Kyle (1985) and Kyle (1989)). The model is closely related to models of dynamic trading with strategic traders (e.g., Vayanos (1999), Rostek and Weretka (2015), Kyle et al. (2017), Du and Zhu (2017), Duffie and Zhu (2017), and Antill and Duffie (2019)). Allocations are inefficient because traders have market power and trade slowly to reduce their price impact. My model builds on the model by Sannikov and Skrzypacz (2016). They develop a tractable framework for dynamic trading with cross-sectional heterogeneity among traders. I analyze heterogeneity in the time-series. The effect on liquidity can be significant because traders strategically choose when to trade. If liquidity needs vary over time, then it may be optimal to reduce liquidity in some periods to reallocate liquidity to periods when liquidity needs are larger. This prediction differs significantly from the welfare implications of Antill and Duffie (2019), where efficient mechanisms at random times reduce welfare. The important assumption that drives the difference between this and their model is the timing of high liquidity periods.

There is a large literature that studies how inventories of market makers affect liquidity going back to at least Garman (1976) (e.g., Amihud and Mendelson (1980), Stoll (1978), Ho and Stoll (1981), Ho and Stoll (1983), and Grossman and Miller (1988)). The effect of modelling dynamic variation in the ability to trade or hold inventories can be much stronger, weaker, or even have the opposite sign compared to a comparative statics analysis.

Several papers model time-varying liquidity in the presence of privately informed investors. Two mechanisms are an option value to wait due to exogenous arrival of news (Daley and Green (2012) and Daley and Green (2016)) or time-varying noise trading intensity (Collin-Dufresne and Fos (2016)), or sentiments (Asriyan et al. (2019)). My model is, to my knowledge, the first model where strategic behavior causes time-variation in liquidity in a setting where inventory concerns drive trading. Brunnermeier and Pedersen (2009), and Kondor and Vayanos (2019) study the dynamics of liquidity when wealth concerns of market makers are important, either through leverage constraints or risk-aversion. One important economic difference between strategic behavior and wealth effects is the duration of liquidity shocks. A market maker may need substantial time to replenish his or her wealth, but strategic behavior can change in a matter of minutes.

The aforementioned papers have endogenous price impacts from trading. A common simplification is to use exogenously specified price impacts (e.g., Acharya and Pedersen (2005) and Collin-Dufresne et al. (2019)). Similarly, the practitioner oriented literature on optimal execution considers the question on how to optimally execute a large order (e.g., Bertsimas and Lo (1998) and Almgren and Chriss

(2001)). My model endogenizes the exogenous specified variation in liquidity. Furthermore, every trader endogenously behaves as if the price follows a process of the form

$$P_t = P_0 + \sigma B_t + I_t q_t + \int_0^t \Lambda_s q_s ds,$$

where q_t is the trading rate at time t . There are two price impacts, one temporary ($I_t q_t$) and one permanent ($\int_0^t \Lambda_s q_s ds$). This specification is closely related to that of Almgren and Chriss (2001).

Chordia et al. (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001), and Chordia et al. (2001) find evidence of time-variation of liquidity and that there is commonality among stocks. Pástor and Stambaugh (2003) and Acharya and Pedersen (2005) study liquidity risk and find support for the hypothesis that liquidity risk affects expected returns. Nagel (2012) relates the return of reversal strategies to time-varying liquidity provision. He shows that these strategies perform well when volatility is high, and liquidity providers are more hesitant to hold inventories. Collin-Dufresne and Fos (2015) study how informed, strategic traders time their trading behavior. Investors choose to trade more in periods when they obtain a lower price impact, which is consistent with the behavior of strategic traders in my model.

2.4 Model setup

The setup builds on the model by Sannikov and Skrzypacz (2016) with an important difference. Their model has traders that are heterogeneous in the cross-section, but homogeneous in the time-series. I study the opposite; traders that are homogeneous in the cross-section, but heterogeneous in the time-series.

Agents: There are $n \geq 3$ traders who receive inventory shocks and trade to reduce their inventory costs. The utility of trader i is given by

$$E \left[\int_0^\infty e^{-rt} \left(-\frac{b(s_t)(X_t^i)^2}{2} + p_t q_t^i \right) dt \right],$$

where $b(s_t)$ is the state-dependent inventory cost, p_t is the price, and q_t^i is the trading rate of trader i at time t ³. That is, traders maximize trading revenue net of quadratic inventory costs. Quadratic inventory costs is a reduced form specification to capture aspects such as risk-aversion, leverage constraints, or funding costs while retaining a tractable linear equilibrium⁴. Inventories follow

$$dX_t = \sigma(s_t) dB_t - q_t dt,$$

where $\sigma(s_t)$ is a state-dependent constant, B_t is a vector of n independent Brownian motions, X_t is

³An equivalent setup is one without discounting, but where the trading horizon is random and exponentially distributed with expected value $\frac{1}{r}$. See Antill and Duffie (2019) for an analysis of this problem. Both alternatives have the same price and trading rates in equilibrium, but calibrated values of r can differ between them. I.e., a calibrated value of $r = 1$ is not very realistic as a discount rate, but may be a realistic trading horizon.

⁴Sannikov and Skrzypacz (2016) solve a microfounded model where traders have exponential utility, and the asset has a risky stream of dividends. The microfounded model adds one additional term to the equilibrium conditions to account for the risk of inventory shocks. This model does not have any nice, closed-form solutions, but numerical examples do not differ much from the reduced form alternative.

the vector of inventories, and q_t is the vector of trading intensities.

Time-variation: Time-variation in liquidity is captured by a state s_t . I assume that there is a random state, $s_t \in \{s_1, s_2\}$, that changes with intensity $\lambda(s_t)$ ⁵. This setup allows me to investigate the impact of dynamic shocks to both the model parameters and the composition of active traders on equilibrium behavior. I allow for variation between the states of the following form:

- The number of active traders differ between the states. All traders can trade in state s_1 , whereas $m \leq n$ of the traders can trade in state s_2 . Exactly which of the traders who can trade in state s_2 is random with equal probability⁶. The equilibrium behavior is similar to a setting where the m traders who can trade in state s_2 is always the same. This problem is significantly harder to solve and may require stronger model assumptions and numerical solution techniques. For example, it is possible to solve it with the conditional double-auction used by Sannikov and Skrzypacz (2016), and numerical solutions suggest that the results are in line with the specification I use in this chapter.
- The inventory cost parameter $b(s)$ may vary between states.

Information structure: Traders are privately informed about their inventories, X_t^i , which is what causes the trading friction. They update their beliefs about the inventories of the other traders. If some traders are restricted in state s_2 , then I assume that the total inventories of the traders who are restricted in state s_2 is observable at the time of the state change from state s_1 to state s_2 . This assumption is to avoid a complex problem where traders can alter other traders' beliefs about the inventories of inactive traders by out-of-equilibrium behavior⁷. In practice, this is perhaps not a strong assumption. Traders may want to clear their books of unwanted assets before leaving the market, in which case the unwanted inventories of inactive traders should be close to 0.

Trading mechanism: The trading mechanism is a uniform-price double auction. Unrestricted traders conjecture that other unrestricted traders submit linear supply-demand schedules of the form

$$q_t^i = \alpha_0 + \alpha_1 p_t + \alpha_2 X_t^i$$

in equilibrium. I restrict attention to *symmetric, acceptable, Markov perfect* equilibria. That is, all traders use the same parameters, α_1 and α_2 only depend on the current state s_t and α_0 depends on both the current state and the sum of restricted traders' expected inventories. An equilibrium is *acceptable* if $\alpha_1 < 0$ and $\alpha_2 > 0$. The market clears at every time t , $\sum_i q_t^i = 0$.

Solution technique: Solving for an equilibrium in the uniform-price double auction is cumbersome. Following Sannikov and Skrzypacz (2016), I implement the equilibrium with a direct revelation mechanism where all unrestricted traders report their inventories. The direct revelation mechanism is characterised by a matrix $Q(s)$ and a vector $P(s)$ such that $q_t = Q(s_t)\hat{X}_t$ and $p_t = P(s_t)E[\hat{X}_t]$. Here, \hat{X}

⁵That is, $Probability(s_{t+dt} = s_2 | s_t = s_1) = \lambda(s_1)dt$ and $Probability(s_{t+dt} = s_1 | s_t = s_2) = \lambda(s_2)dt$.

⁶It is an abuse of notation to define s_t as two states in this setting. There are $1 + \binom{n}{m}$ states, but two aggregate states. It is nevertheless sufficient to solve for an equilibrium since every trader only cares if he or she is restricted or not conditional on their beliefs about other traders' inventories.

⁷This assumption can be relaxed if the market is augmented with size-discovery mechanisms of the type analyzed by Antill and Duffie (2019) at the time of state-changes.

is a vector of reports of the active traders and the expected inventories of inactive traders. Inactive traders do not report their inventories, but the expected value of their inventories affects the price. The structure of $Q(s)$ and $P(s)$ is as follows: $Q(s)$ has $q(s)$ on the diagonal positions for unrestricted traders, $-\frac{q(s_1)}{n-1}$ or $-\frac{q(s_2)}{m-1}$ on the off-diagonal position for unrestricted traders to ensure market clearing, and 0 on all rows and columns for inactive traders. $P(s)$ has $p^u(s)$ on the positions for unrestricted traders and $p^r(s)$ on the positions for restricted traders. A direct revelation mechanism is *acceptable* if $p^u < 0$ and $q(s) > 0$.

Lemma 5. *If there is a linear, acceptable, and Markov perfect equilibrium in the double auction, then there is a corresponding acceptable direct revelation mechanism that is truth-telling, and vice versa. Every trader can infer the sum of other active traders' inventories in equilibrium.*

Proof: See Appendix B.1.1.

2.5 Model solution

To solve for an equilibrium, we need to find trading matrices $Q(s)$ and price vectors $P(s)$ such that truth-telling is optimal in the direct revelation mechanism. For every trader, there are three state-variables, $Y_{it} = E[X_t | \mathcal{F}_{it}]$, s_t , and $\gamma_{it} = E[(X_t - Y_{it})(X_t - Y_{it})^T | \mathcal{F}_{it}]$. That is, each trader potentially cares about the first two moments of inventories for all traders, and the current state. Trader i announces $\hat{X}_t^i = X_t^i + y_t^i$ at time t and all reports are truthful in equilibrium, i.e. $y_t^i = 0$. Define the value function of trader i as

$$f^i(Y_{it}, s_t, \gamma_{it}) = \sup_{y^i} E \left[\int_t^\infty e^{-r(u-t)} \left(-\frac{b(s_u)(X_u^i)^2}{2} + p_u q_u^i \right) du \middle| \mathcal{F}_{it} \right].$$

The solution to the maximization problem has to satisfy the HJB-equation

$$r f^i(Y_{it}, s_t, \gamma_{it}) = \sup_{y_t^i} -\frac{b(s_t)(X_t^i)^2}{2} + p_t q_t^i + \mathcal{A}(s_t, y_t^i) f^i(Y_{it}, s_t, \gamma_{it}) \quad (2.1)$$

where the infinitesimal generator is defined as

$$\mathcal{A}(s_t, y_t^i) f^i(Y_{it}, s_t, \gamma_{it}) = \lim_{h \downarrow 0} \frac{E[f^i(Y_{i(t+h)}, s_{t+h}, \gamma_{i(t+h)}) | \mathcal{F}_{it}, y_t^i] - f^i(Y_{it}, s_t, \gamma_{it})}{h}.$$

I solve the maximization problem in three steps; the first step is to find the functional form of the value function, the second step is to solve the filtering problem, and the last step is to solve the maximization problem. I use the following notation in the derivation: $Q^i(s)$ as the i -th column and $\underline{Q}^i(s)$ as the i -th row of $Q(s)$. The remaining notation is as described above. The following Lemma gives the functional form of the value function:

Lemma 6. *The value function of trader i has the functional form*

$$f^i(Y_{it}, s_t, \gamma_{it}) = Y_{it}^T A^i(s_t) Y_{it} + k^i(s_t) + \text{tr}[A^i(s_t) \gamma_{it}]$$

where $A^i(s)$ is a state-dependent symmetric matrix and $k^i(s)$ is a state-dependent constant.

Proof: see Appendix B.1.2.

There are two elements in Lemma 6 that depend on the trading behavior and inventory shocks, the matrices $A^i(s)$ and the constants $k^i(s)$. What is also clear, is that we can analyze the value function for a trader who always observes X_t with the same price and inventory dynamics as in the equilibrium. This simplifies the functional form to

$$\tilde{f}^i(X_t, s_t) = X_t^T A^i(s_t) X_t + k^i(s_t)$$

and it has to satisfy the differential equation

$$\begin{aligned} r \tilde{f}^i(X_t, s_t) = & -\frac{b(s_t)(X_t^i)^2}{2} + P(s_t) X_t \underline{Q}^i(s_t) X_t - \tilde{f}_X^i(X_t, s_t) Q(s_t) X_t \\ & + \frac{\sigma^2(s_t)}{2} \text{tr}[\tilde{f}_{XX}^i(X_t, s_t)] + \lambda(s_t) \left(E[\tilde{f}^i(X_t, s')] - \tilde{f}^i(X_t, s_t) \right). \end{aligned}$$

The differential equation comes from equation (2.1) when all traders behave as in equilibrium (i.e., $y^i = 0$). The term $P(s_t) X_t$ is the price and the term $\underline{Q}^i(s_t) X_t$ is the trading rate of trader i . The state s' is the state after the next state change. Together with the functional form, this simplifies to

$$\begin{aligned} (r + \lambda(s)) \left(X^T A^i(s) X + k^i(s) \right) = & -\frac{b(s)(X^i)^2}{2} + P(s) X \underline{Q}^i(s) X \\ & - 2X^T A^i(s) Q(s) X + \sigma^2(s) \text{tr}[A^i(s)] \\ & + \lambda(s) \left(X^T E[A^i(s')] X + E[k^i(s')] \right). \end{aligned} \quad (2.2)$$

Equation 2.2 needs to hold for all traders in both states and for all X . This gives the following sets of equations:

$$\begin{aligned} (r + \lambda(s)) A^i(s) = & -\frac{b(s)}{2} \mathbf{1}^{ii} + \frac{P^T(s) \underline{Q}^i(s) + (\underline{Q}^i(s))^T P(s)}{2} \\ & - A^i(s) Q(s) - Q^T(s) A^i(s) + \lambda(s) E[A^i(s')]. \end{aligned} \quad (2.3)$$

and

$$(r + \lambda(s)) k^i(s) = \sigma^2(s) \text{tr}[A^i(s)] + \lambda(s) E[k^i(s')]. \quad (2.4)$$

We need to solve the system of equations (2.3) and (2.4) for both states. If some traders are restricted in state s_2 , then we need to solve the system of equations for both restricted and unrestricted traders in state s_2 . The expectations, $E[A^i(s')]$ and $E[k^i(s')]$, account for the possibility that some traders are restricted in state s_2 , in which case there are matrices $A^i(s_2)$ and constants $k^i(s_2)$ for restricted and unrestricted traders.

Before solving the maximization problem, we have to analyze the filtering problem to get the dynamics of Y_{it} and γ_{it} .

Lemma 7. *If all traders except trader i report truthfully and trader i reports $\hat{X}_t^i = X_t^i + y_t^i$, then*

$$E[dY_{it} | \mathcal{F}_{it}] = -(Q(s_t) Y_{it} + Q^i(s_t) y_t^i) dt.$$

Both $d[Y_i^j, Y_i^k]_t$ and $E[d\gamma_{it}]$ are independent of y^i where $d[Y_i^j, Y_i^k]_t$ is the quadratic covariation

between Y_i^j and Y_i^k .

Proof: see Appendix B.1.3.

Lemma 7 is important for the maximization problem. It shows that trader i 's behavior has an impact on Y_{it} , but not on γ_{it} . This means that all terms related to γ_{it} vanishes in trader i 's first order condition related to y^i . Furthermore, since all traders can infer the sum of inventories of other trader in equilibrium (Lemma 5), we have $P(s)X_t = P(s)Y_{it}$ and $\underline{Q}^i(s)X_t = \underline{Q}^i(s)Y_{it}$ in equilibrium. When we analyze the maximization problem for unrestricted traders,

$$\sup_{y^i} \frac{-b(s)(Y_i^i)^2}{2} + (P(s)Y_i + p^u(s)y^i)(\underline{Q}^i(s)Y_i + q(s)y^i) + \mathcal{A}(s, y^i) f^i(Y_i, s, \gamma_i),$$

the only element in $\mathcal{A}(s, y^i)$ that depends on y^i is the drift term of Y_i , which is $-(Q(s)Y_i + Q^i(s)y^i)dt$. The first order condition is

$$p^u(s)\underline{Q}^i(s)Y_i + P(s)Y_i q(s) + 2p^u(s)q(s)y^i - 2Y_i^T A^i(s)Q^i(s) = 0$$

where $\frac{\partial f(Y_i, s, \gamma_i)}{\partial Y_i} = 2Y_i^T A^i(s)$. In equilibrium, we have truth-telling ($y^i = 0$) and the first-order conditions must hold for all Y_i . This gives the system

$$p^u(s)\underline{Q}^i(s) + q(s)P(s) = 2(A^i(s)Q^i(s))^T. \quad (2.5)$$

The second-order condition for a maximum is $p^u(s)q(s) < 0$, which is satisfied for an *acceptable* direct revelation mechanism.

We need to solve the system of equations from (2.3), (2.4), and (2.5) to find $Q(s)$, $P(s)$, $A^i(s)$, and $k^i(s)$ for both active and inactive traders and for both states. The uncertainty about X_t , captured by γ_{it} , affects welfare but not the trading behavior or price dynamics.

Proposition 4. *If $(Q(s), P(s), k^i(s)$ and $A^i(s)$, $i = 1, \dots, N, s = s_1, s_2$) solves the system (2.3)-(2.5), and the direct revelation mechanism is acceptable, then all traders prefer to tell the truth rather than any other action that satisfies the no-Ponzi condition $E[e^{-rt} X_t^2] \rightarrow 0$.*

Proof: See Appendix B.1.4.

In practice, traders care about their price impact when they make their trading decisions. The equilibrium from Proposition 4 maps into a setting where traders have a temporary and a permanent price impact⁸. First, in equilibrium, trader i trades smoothly at rate q_t^i and price is given by

$$p_t = \bar{p}_t^i + I(s_t)q_t^i.$$

Here, \bar{p}_t^i is a history- and state-dependent price level and $I(s)$ is the *temporary price impact* defining the sensitivity of prices to the instantaneous trading rate. As in Sannikov and Skrzypacz (2016), my

⁸The naming conventions about price impacts are ambiguous. I follow Kyle et al. (2017) and Sannikov and Skrzypacz (2016). This distinction between the temporary and permanent price impacts is similar to the literature on optimal execution (e.g., Almgren and Chriss (2001) and Huberman and Stanzl (2004)). The empirical market microstructure literature has a slightly different interpretation of a temporary price impact (e.g., the bid-ask bounce).

model also features a permanent price impact $\Lambda(s)$ ⁹ that determines the reaction of equilibrium prices to large out-of-equilibrium trades.

Formally defining the permanent price impact requires a bit of work. Consider an (out-of-equilibrium) scenario whereby a large trader evaluates the consequence of trading a discrete quantity of x units of the asset over a short period $[t_0, t_0 + k]$ by increasing the trading rate in excess of the equilibrium trading rate q_t^i , so that the realized trading rate is given by

$$\tilde{q}_t^i = \begin{cases} q_t^i, & t < t_0 \\ q_t^i + x/k, & t \in [t_0, t_0 + k] \\ q_t^i, & t > t_0 + k. \end{cases}$$

A short interval, $[t_0, t_0 + k]$, is to ensure that the trader is able to perform this out-of-equilibrium behavior before the next state change, after which the trader risk to be restricted. The out-of-equilibrium behavior will affect the price in the following way: at time t_0 there will be a discrete jump in the price equal to $I(s) \frac{x}{k}$. The price will gradually increase between time t_0 and $t_0 + k$ before there is a new discrete jump down at time $t_0 + k$. The formal definition of the *permanent price impact*, $\Lambda(s)$, is

$$\lim_{k \downarrow 0} E_{t_0} [P_{t_0+k} - P_{t_0}] = \Lambda(s)x.$$

The explicit expressions for $I(s)$ and $\Lambda(s)$ can be derived in terms of the equilibrium quantities $P(s)$ and $Q(s)$.

Corollary 5. *The temporary and permanent price impacts are given by $I(s) = -\frac{p^u(s)}{q(s)}$, $\Lambda(s_1) = -p^u(s_1) \frac{n}{n-1}$, and $\Lambda(s_2) = -p^u(s_2) \frac{m}{m-1}$.*

Proof: See appendix B.1.5.

The equilibrium trading rate affects the temporary price impact because it is informative about future trading intentions. The permanent price impact is related to the capacity to hold inventories. This difference offers testable predictions of the model as some shocks have a significant effect on the equilibrium trading intensity, but not on the capacity to hold inventory and vice versa.

Figure 2.1 shows the expected effect on the price and inventories when a trader deviates and buys at a fixed rate over a short period. During the time the trader buys, there is a temporary price impact. When he or she stops buying, the price immediately jumps down, and only the permanent price impact is left.

In practice, the way traders think about their price impact is not by solving dynamic market-microstructure models. Instead, they use their available data to analyze how different execution strategies affect the cost of trading. The thought experiment with out-of-equilibrium behavior will likely give similar results as that of a trader who analyzes the price impact of their previous trades. Rostek and Weretka (2015) emphasize this kind of behavior and use an equilibrium concept where traders trade against their price impact.

⁹There is also a transient price impact in the model by Sannikov and Skrzypacz (2016). This price impact is due to heterogeneity in the inventory cost parameter b between traders in their model. There is no heterogeneity in the cross-section of inventory cost parameters in my model. Hence, there is no transient price impact.

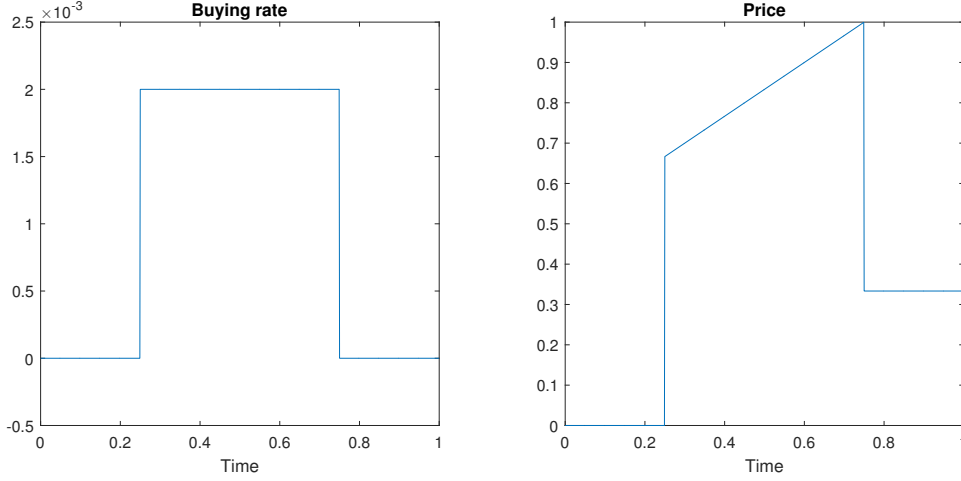


Figure 2.1 – The figure shows the effect on the price when a trader buys one unit of the asset in excess of the equilibrium behavior.

2.6 Time-varying liquidity

If traders have constant inventory cost and can trade all the time, then there is a simple, closed-form solution to the equilibrium conditions.

Result 1. (Sannikov and Skrzypacz, 2016): *The unique linear symmetric equilibrium without time-variation is characterised by:*

$$P = -\frac{b}{nr}\underline{1} \text{ and } Q = \frac{n-2}{2}r\left(I - \frac{S}{n}\right)$$

and matrices A^i with

$$a_{ii} = -\frac{b}{2r} \frac{3n-2}{n^2}, a_{ij} = -\frac{b}{2r} \frac{n-2}{n^2} \text{ and } a_{jk} = \frac{b}{2r} \frac{n-2}{(n-1)n^2}$$

where $\underline{1}$ is a vector with ones and S is an $n \times n$ matrix with ones.

Result 1 is useful as a benchmark to understand how explicitly modelling time variation affects the equilibrium. If parameters are constant, then the price impacts are $I = \frac{2b}{(n-2)(n-1)r^2}$ and $\Lambda = \frac{b}{(n-1)r}$. Fewer traders always increases both price impacts, but there has to be a large drop in the number of traders to significantly increase them. This is no longer true when variation in n is explicitly modelled. The inventory cost parameter, b , always enters linearly in both price impacts. When variation in b is modelled explicitly, the temporary and permanent price impacts may even be negatively correlated due to equilibrium behavior. I analyze the impact of time-variation in each of the two elements separately to preserve tractability.

2.6.1 Time-varying competition

Traders may leave the market after large losses as in the case of Knight Capital, enter when investment opportunities improve, or only be present episodically to reduce costs relative to continuous presence. In this section, I show that such behavior can have a substantial effect on liquidity.

The system of equation from (2.3)-(2.5) reduces to a cubic equation, (B.4), for the trading rate $q(s_2)$ in the constrained state. The following is true:

Proposition 5. *There exists a one-to-one correspondence between solutions to the cubic equation (B.4) satisfying the second order conditions $p^u(s_k)q(s_k) < 0$ for $k = 1, 2$ and linear, symmetric equilibria with trade in both states. A sufficient condition for the existence of a symmetric equilibrium with trade in both states is that $\lambda(s_2) < \bar{\lambda}$ where $\bar{\lambda} = \frac{rn(n-1)}{2} \frac{m-2}{n-m}$.*

Proof: see Appendix B.1.6.

The second-order conditions hold in both states if there exists a positive solution to the cubic equation (B.4) that is not too large¹⁰. To understand the equilibrium behavior, we can look at the two solutions where one state is absorbing.

Corollary 6. *If $\lambda(s_2) = 0$, then the equilibrium trading rates and prices are given by*

$$\begin{aligned} q(s_1) &= \frac{(n-1)(n-2)r}{2n} + \lambda(s_1) \frac{(n-m)(mn-2)}{2mn} \\ q(s_2) &= \frac{(m-1)(m-2)r}{2m} \\ p^u(s_1) &= -\frac{b}{nr} \\ p^u(s_2) &= -\frac{b}{mr} \\ p^r(s_2) &= 0. \end{aligned}$$

If $\lambda(s_1) = 0$, then

$$\begin{aligned} q(s_1) &= \frac{(n-1)(n-2)r}{2n} \\ q(s_2) &= \frac{(\lambda(s_2) + r)m((n-1)(m-2)nr - 2\lambda(s_2)(n-m))}{2m(\lambda(s_2)((n-2)m+n) + rn(n-1))} \\ p^u(s_1) &= -\frac{b}{nr} \\ p^u(s_2) &= -\frac{b}{nr} \left(1 + \frac{(n-m)(\lambda(s_2) + (n-1)r)}{m(n-1)(\lambda(s_2) + r)}\right) \\ p^r(s_2) &= -\frac{b}{nr} \frac{(n-2)\lambda(s_2)}{(n-1)(\lambda(s_2) + r)}. \end{aligned}$$

Proof: see appendix B.1.7.

Numerical solutions with $\lambda(s) > 0$ for both states have the same implications as the solutions of Corollary 6. The effect of time-varying number of traders is that the trading intensity increases in state s_1 . The trading intensity in state s_2 is non-linear in $\lambda(s_2)$ except for $n = 4$ and $m = 3$, but always lower than the trading intensity in state s_1 . There are two factors that affect the trading intensity in state s_2 , the option value to wait and $p^u(s_2)$. If $\lambda(s_2) > 0$, then $|p^u(s_2)|$ is lower than what it would have

¹⁰I conjecture that there is a unique linear, symmetric equilibrium with trade in both states if $\lambda(s_2) < \bar{\lambda}$ and no such equilibria if $\lambda(s_2) \geq \bar{\lambda}$. It is straightforward to prove the conjecture for specific values for n , m , and $\lambda(s_1)$.

Table 2.1 – The table shows the differences in the temporary and permanent price impacts for different numbers of traders. The last two rows illustrate the comparative statics result. The comparative statics result is the change in the price impact from an unexpected, permanent decrease from N to m traders. The values for b has no effect on the ratio of price impacts and $r = 0.05$.

(n,m)	(20,19)	(100,95)	(1000,950)	(20,19)
$(\lambda(s_1), \lambda(s_2))$	(52, 52)	(52, 52)	(52, 52)	(12, 52)
Ratio temporary price impact	2.036	1.077	1.051	1.621
Ratio permanent price impact	1.006	1.001	1.000	1.006
Comparative statics ($\lambda(s_i) = 0$)				
Ratio temporary price impact	1.110	1.109	1.107	1.110
Ratio permanent price impact	1.055	1.052	1.052	1.055

been if the state was always s_2 , which reduces the incentives for waiting. The two factors have the opposite effect on the trading rate in state s_2 , but the option value to wait always dominates for large values of $\lambda(s_2)$. The temporary price impact $I(s_2) = -\frac{p^u(s_2)}{q(s_2)}$ (see Corollary 5) in state s_2 increases in $\lambda(s_2)$. This means that short-lived shocks have a larger effect on the temporary price impact than persistent shocks. The opposite is the case for the permanent price impact, which depends on $p^u(s_2)$ and is decreasing in $\lambda(s_2)$. The permanent price impact is constant in state s_1 whereas the temporary price impact is decreasing in $\lambda(s_1)$ and increasing in $\lambda(s_2)$. The reason is that a larger value for $\lambda(s_1)$ reduces the expected time until the state changes to state s_2 whereas a larger value for $\lambda(s_2)$ reduces the expected time in state s_2 . Both values affect the incentives to trade faster with better liquidity today.

Table 2.1 shows the price impacts for calibrated versions of the model. All examples use a discount rate of $r = 5\%$ and assume that 5% of the traders' population are restricted in state s_2 . The state switches on average once a week in the first three columns. The fourth column has different expected lengths of the time in each state with state s_1 lasting one month on average. The permanent price impact is almost constant in all examples, whereas the temporary price impact may have strong time-variation even from small changes in the number of traders. The reason is that the option value of waiting dominates when the amount of competition is low or if the state changes frequently. If the number of traders is large, then the effect is approximately the same percentage difference as the difference in the number of traders. The intuition for this is that when there is a large number of traders in the market, then the inefficiencies in allocations are traded away quickly, and the incentives to wait for better liquidity are less important. The empirical prediction is that liquidity varies more or breaks down more frequently in markets that are dominated by a few large traders than in markets with many traders. The last two rows show the change in the price impact from comparative statics analysis on the results when all traders trade all the time (Result 1). The comparative statics effects differ significantly from my results, which shows that explicitly modeling the composition of traders is essential in settings where the composition changes over time.

Some markets, such as the corporate bond market, are dominated by a few large investors. The numerical examples suggest that these markets should be more prone to large reductions in liquidity. Ivashchenko and Neklyudov (2018) find that a significant fraction of corporate bonds experience both periods where they are frequently and infrequently traded. Their results suggest that the frequency of trade for these bonds is driven by variation in dealer behavior. Other markets, such as the E-mini S&P 500 futures, have a large number of active traders and should experience less variation in liquidity from

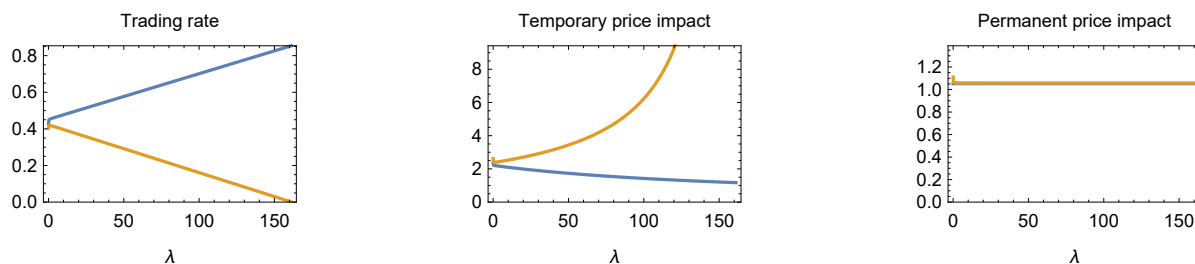


Figure 2.2 – The plots show trading rates and price impact when $n = 20$ and $m = 19$, $r = 0.05$ and $\lambda(s_1) = \lambda(s_2) = \lambda$ for different values of λ . Yellow lines are the values for state s_1 and blue for state s_2 .

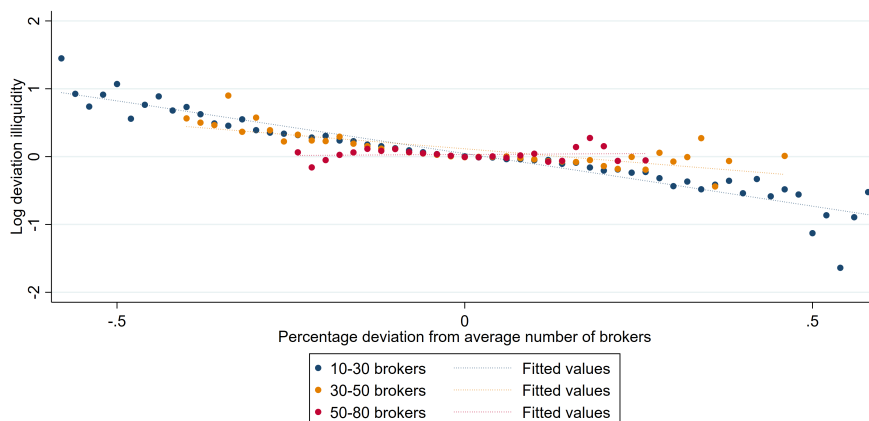


Figure 2.3 – The figure shows the relationship between liquidity, measured by the Amihud measure, and the number of brokers for a firm. The x-axis shows the percentage deviation of average monthly brokers relative to the yearly average for every firm. The y-axis is the logarithm of the monthly deviation relative to the annual value of the Amihud measure. The observations have been averaged in bins for every 0.02 points along the x-axis for each group. The group with few brokers have 10-30 on an average day, the intermediate group has 30-50, and the group with many brokers have 50-80 on average. The sample consists of all NASDAQ stocks with a price between 5\$ and 1000\$, where I have enough information to calculate the average number of monthly brokers for the stock and the Amihud liquidity measure. The period is from the start of 1998 to the end of 2018.

the mechanism in this section. There is one exception if shocks to liquidity are extremely short-lived, then there may also be significant effects in highly liquid markets. One such example is the 2010 Flash Crash, which lasted for less than an hour.

Figure 2.2 shows how shocks of different expected duration affect liquidity. Only short-lived shocks have a large effect on liquidity. This result is in line with the liquidity measure proposed by Pástor and Stambaugh (2003). Large deviations from average liquidity tend to be short-lived, except for the months after the financial crisis. Short-lived, in my model, means that traders expect to switch to the other state before the inefficiencies are traded away. The empirical prediction is that drops in liquidity are larger when traders have the option to wait. One example is from derivative markets, where the time to maturity may be a proxy for the option value to wait. If liquidity drops for some exogenous reason, then we would expect to see a larger drop in liquidity for derivatives with a long time to maturity.

Figure 2.3 shows the relationship between liquidity, as measured by the Amihud measure and the number of brokers actively trading for different firms listed on Nasdaq. I use deviations from the

average, annual values to adjust for time-trends. There is a clear, negative relationship for the firms with few brokers. The relationship is weaker for firms with more brokers and close to 0 for the firms with the largest number of brokers. This result is in line with the predictions of this section; the number of brokers affect liquidity, but mainly for the firms with few brokers.

2.6.2 Time-varying inventory costs

If all traders always trade and the inventory cost parameter $b(s)$ differ between states, then the unique linear, symmetric equilibrium is:

Proposition 6. *The unique linear, symmetric equilibrium has trading rates and price given by $q_t = Q(s_j)X_t$ and $p_t = P(s_j)X_t$ with*

$$Q(s_j) = \frac{n-2}{n} r \left(I - \frac{S}{n} \right) \left(\frac{\lambda(s_j) + \lambda(s_k) + r}{\frac{b(s_k)}{b(s_j)} \lambda(s_j) + \lambda(s_k) + r} \right)$$

$$P(s_j) = -\frac{1}{nr} \underline{1} \left(\frac{b(s_j)(\lambda(s_k) + r)}{\lambda(s_j) + \lambda(s_k) + r} + \frac{b(s_k)\lambda(s_j)}{\lambda(s_j) + \lambda(s_k) + r} \right).$$

Here S is an $n \times n$ matrix with 1 in every position and $\underline{1}$ is an n -dimensional vector with 1 in every position.

Proof: see Appendix B.1.8.

Without loss of generality, I assume that $b(s_1) < b(s_2)$. In this case, the equilibrium is characterized by a lower trading intensity and a higher permanent price impact in state s_1 compared to a setting where the state is always s_1 . The opposite is true for state s_2 . The difference in the temporary price impacts is ambiguous. The reason is that the temporary price impact is given by the quotient $I(s) = -\frac{p^u(s)}{q(s)}$. In equilibrium, both inequalities $|p^u(s_1)| < |p^u(s_2)|$ and $q(s_1) < q(s_2)$ hold, and thus the total effect depends on which component dominates. The dominating effect depends on the parameters of the model in the following way:

Corollary 7. *The temporary price impact is higher in state s_1 if*

$$r < \sqrt{\frac{b(s_1)}{b(s_2)}} \left(1 + \frac{b(s_2)}{b(s_1)} \frac{\lambda(s_1)}{\lambda(s_2)} \right) \lambda(s_2),$$

and is higher in state s_2 if the inequality is reversed.

Proof: see Appendix B.1.9.

Nagel (2012) links reversal strategies to liquidity provision. He argues that market makers are more hesitant to hold inventories (large $b(s)$) when volatility is high and shows that reversal strategies perform better in periods with high volatility. The illiquidity measure of Pástor and Stambaugh (2003) is defined as an estimate of the temporary effect of volume on returns. Hence, it can be naturally interpreted as measuring the temporary price impact. By contrast, the measure in Nagel (2012) captures the response of prices to inventory shocks and can be interpreted as an estimate of

permanent price impact¹¹.

Perhaps surprisingly, these two illiquidity measures have a low correlation. This finding is hard to rationalize in standard models where the two price impacts (temporary and permanent) tend to move in the same direction. By contrast, I can easily explain this finding in my model because the endogenous timing decisions of strategic traders decouple the equilibrium dynamics of the two price impacts, as is illustrated by Proposition 5 and Corollary 7. Shocks to $b(s)$ affect the equilibrium impact of inventory imbalances through $p(s)$, but may not affect the temporary price impact much if $r \approx \sqrt{\frac{b(s_1)}{b(s_2)}} \left(1 + \frac{b(s_2)}{b(s_1)} \frac{\lambda(s_1)}{\lambda(s_2)}\right) \lambda(s_2)$. As shown in the previous section, a temporary shock to competition has the opposite effect.

Bogousslavsky and Collin-Dufresne (2019) find that liquidity and volume are negatively correlated for small and medium-sized stocks, as predicted by models of asymmetric information. Large stocks exhibit an opposite behavior, whereby liquidity and volume are positively correlated. Corollary 7 provides an alternative explanation for this difference in behavior. Namely, trading volume is larger in state s_2 , but the temporary price impact can be larger in either state. They will be negatively related if $\frac{b(s_2)}{b(s_1)}$ is sufficiently large or if r is sufficiently low. If there is more variation in the cost of holding inventories for small stocks than for large stocks, for example, through a flight to quality/liquidity mechanism, then the model gives predictions in line with the empirical observations.

2.7 Welfare and time-varying number of traders

In this section, I study the effect of a time-varying composition of traders on welfare. The welfare of trader i is defined as his or her trading revenue net of inventory costs. In the setting where $b(s) = b$ and the number of traders is constant, it is always better to have more traders to reallocate inefficiencies faster. In this section, I show that it can be welfare-improving to restrict some of the traders in one of the states if the magnitude of inventory shocks is also time-varying. In particular, I show how changes in the composition of traders can endogenously move liquidity to the state with larger liquidity needs.

Let $\sigma^2(s_1)$ and $\sigma^2(s_2)$ capture the time-variation in liquidity needs. All other choices are as in Proposition 5. There are three elements that affect welfare, the current inventories, future inventory shocks, and the uncertainty about current inventories. To make the analysis tractable, I analyze welfare when $X_0 = 0$. The difference in welfare is captured by the difference in the constants $k^i(s)$.

Proposition 7. *Suppose that $\sigma^2(s_1)$ is sufficiently large relative to $\sigma^2(s_2)$, $0 < \lambda(s_1)$, $0 < \lambda(s_2) < \bar{\lambda} = \frac{rn(n-1)}{2} \frac{m-2}{n-m}$, and that the inventory cost parameter $b(s)$ is the same in both states. Consider two equilibria: The unconstrained equilibrium (in which all traders participate in both states) and a constrained equilibrium in which a random group of $n - m$ traders is restricted from participation in state s_2 . Then, assuming zero initial inventories X_0 , every single trader is strictly better off in the constrained equilibrium in every single state. That is, $f_{constrained}^i(0, s_k, 0) > f_{unconstrained}^i(0, s_k, 0)$ for every i and every k .*

Proof: see Appendix B.1.10.

¹¹Inventory shocks are permanent in my model, and hence there is no return reversal. If aggregate inventory shocks revert to 0 as in models such as the one by Grossman and Miller (1988), then return reversal would be a measure of Λ_t . One simple way to model this is to let aggregate inventories evolve according to $dX_t = -q_t dt + \sigma(s_t) dB_t - \phi X_t dt$. The results of this section still holds with this specification, but the discount rate r has to be replaced by $r + 2\phi$.

The welfare result of Proposition 7 is striking. Every trader could be better off if he or she could commit to not always trade, as long as some other traders still do. However, since they cannot commit, there is a place for welfare-improving policies. To the best of my knowledge, no other model of strategic trading can produce such a strong welfare result. For example, Malamud and Rostek (2017) find that, in a static model with many partially fragmented markets, restricting participation in some markets improves total welfare, yet some traders are always worse off. By contrast, Proposition 7 shows how all traders could be better off in constrained equilibria.

Restricting traders in some states acts as a mechanism for reallocating liquidity across time. Traders anticipating future improvements in liquidity respond by trading less today, amplifying liquidity fluctuations. This result relates my model to that of Antill and Duffie (2019) who introduce size-discovery mechanisms for reallocating liquidity across time. Antill and Duffie (2019) show that such mechanisms (for example, dark pools in equity markets or workups in Treasury markets), always reduce welfare compared to a market with continuous double-auctions. My analysis implies that the result of Antill and Duffie (2019) depends crucially on their assumption of purely random timing of liquidity improvements (namely, Antill and Duffie (2019) assume that size discovery happens at random times unrelated to traders' liquidity needs). If the times of liquidity improvements correlate with traders' liquidity needs (more liquidity at times of more need for liquidity), Proposition 7 shows that this can be welfare improving.

This result has important implications for market regulation: Efficient timing of mechanisms is crucial for their ability to improve allocational efficiency. For example, regulators and market participants are often concerned that large traders, such as high-frequency traders or market makers in general, may restrain from providing liquidity in volatile times (e.g., this is what happened during the *Black Monday*, the crash of 1987). By contrast, Proposition 7 shows that it might be beneficial to restrict some traders in calm times if they voluntarily provide liquidity in times of a storm. Empirical evidence suggests that, indeed, periods with large inventory shocks are also periods when liquidity providers can earn substantial returns, as shown by Nagel (2012). Anand and Venkataraman (2016) study the behavior of endogenous liquidity providers in equity markets. Their results suggest that more liquidity providers are active for a given stock when its volatility is high. In my model, volatility comes from inventory shocks and is linear in $\sigma(s)p^u(s)$. Their results suggest that the market may endogenously behave as in Proposition 7.

2.8 Conclusion

The main new element of my model is the introduction of time-variation in the ability to trade and hold inventories. Strategic traders have an incentive to choose both how much and when to trade. Time-varying conditions for traders offer several new insights with empirical predictions. Variation in the composition of active traders can cause significant differences in liquidity, but only if shocks are short-lived. The relation between measures of liquidity and the ability to hold inventories is ambiguous. They may have positive, negative, or almost no correlation. These prediction explains the low correlation between some liquidity measures. Both results depend on explicitly modeling time-variation. A comparative statics analysis would find much weaker effects of varying competition and always a positive relationship between the ability to hold inventories and liquidity measures.

The model provides new welfare implications. Restricting some traders can be optimal if liquidity needs vary over time. The reason is that such policies can "move" liquidity to periods with larger liquidity needs. On the other hand, the model also shows that liquidity may evaporate more frequently in markets with few participants. Some regulations, such as the "Volcker Rule," intend to reduce the systemic risk in the financial market by limiting banks' ability to engage in proprietary trading. An unintended consequence could be that fewer large traders increase the risk of large shocks to liquidity, which is another type of systemic risk.

The model does not have asymmetric information about the fundamental value of the asset. While adding such a feature to the model is difficult, the likely implications are straightforward. Traders with information about the fundamental value would prefer to trade on this information in periods with high liquidity. As shown by Collin-Dufresne and Fos (2016), such behavior would dampen the variation in liquidity. As a result, we should expect to see liquidity dynamics more in line with the predictions of my model in markets where inventory concerns mainly drive trading.

I find support for the prediction that the number of active traders affects liquidity in equity markets. The effect is stronger for equities with fewer active traders, as predicted by my model. Most empirical measures of liquidity are constructed by using price and volume data. The results of this chapter suggest that information about liquidity providers can provide additional insights into the effect of liquidity and liquidity risk on asset returns.

3 Anonymity and bid shading

3.1 Abstract

I study the incentives of heterogeneous traders to stay anonymous in financial markets. Traders can hide their full trading intentions by splitting orders into multiple small orders, and by staying anonymous. I show that absent of noise, all traders reveal their identity in equilibrium. If markets are noisy, then traders with a good ability to hold inventories stay anonymous. Anonymous markets improve welfare for a subset of the traders, and the results suggest that total welfare is highest in a fully anonymous market.

3.2 Introduction

Strategic traders spend a significant amount of resources to hide their full trading intentions. In this chapter, I consider a model where traders can hide information about their trading intentions in two ways; splitting large orders into multiple smaller orders and trading anonymously. In equilibrium, traders split large orders into smaller ones but reveal their identity unless the order flow is sufficiently noisy. If the total order flow is noisy, the decision to stay anonymous depends on the amount of noise and heterogeneity in the market. The model differs from standard models of strategic trading by assuming that traders can truthfully reveal their identity, and studies the incentives to do so.

A simple example explains the central intuition of this chapter. Consider a market with two types of agents, patient and impatient traders. The two types differ in only one dimension, the speed at which they trade towards their optimal portfolio. For simplicity, impatient traders adjust their portfolios immediately, whereas patient traders split their orders into smaller pieces to reduce their price impact. The price impact depends on the beliefs about which trader submitted the order as it is informative about subsequent orders. Impatient traders have an incentive to reveal their identity to signal that they are already at their optimal portfolio, and their counterparties should not expect any subsequent orders in the same direction.

The previous literature on anonymity has focused on the distinction between informed and uninformed orders and how uninformed traders can obtain better conditions if they can signal or reveal their type. While this distinction is relevant in many markets, there are large and important markets with little private information about fundamentals. Evidence suggests that trading behavior still has

an important effect on prices (e.g., Lou et al. (2013) for sovereign debt or Evans and Lyons (2002) for foreign exchange markets). The implications of trader anonymity may be different in these types of markets. For example, the adverse selection in this chapter is driven by different abilities to hold inventories rather than the ability to predict future prices.

Most trading occurs on electronic trading venues where anonymity is a design choice. Some alternative designs is to impose anonymity for all traders, make the identity of traders fully transparent, or let individual traders choose to reveal their identity or not. Furthermore, most assets can trade at a range of different trading venues, and each venue can make different choices in terms of anonymity to attract traders. Understanding how anonymity affects trading behavior and allocational efficiency is vital to make sound policy recommendations, and for individual investors to minimize the cost of implementing their trading strategies. Some exchanges, such as the stock exchanges in Helsinki, Oslo, and Stockholm, have introduced and later removed post-trade anonymity after mixed opinions from their members.

One assumption in this chapter is that traders can choose to trade anonymously or not. In practice, most trading platforms specify that traders are either anonymous or not. What traders can do is to choose a trading platform that has the desired amount of anonymity. It is challenging to observe choices to trade anonymously because different trading venues differ in multiple dimensions other than anonymity. One exception is the Toronto Stock Exchange (TSX), where traders can choose to trade as a generic broker, and their true identity is only revealed later. Comerton-Forde et al. (2011) study decisions to trade anonymously on TSX and find that the vast majority of traders on TSX reveal their identity (> 90% of volume). Traders who stay anonymous tend to have a larger price impact. Both findings are in line with the predictions of my model. Linnainmaa and Saar (2012) show that the market makes correct inferences about the type of trader behind an order from observing the broker identity. Their results suggest that traders can, at least partially, reveal their type by trading with a broker whose clients are similar to them.

There is a large literature on strategic trading starting with Kyle (1985) and it includes both static (e.g., Kyle (1985) and Kyle (1989)) and dynamic models (e.g., Kyle (1985), Back (1992), Vayanos (1999), Du and Zhu (2017), Duffie and Zhu (2017) and Antill and Duffie (2019)). While static models can account for heterogeneity between market participants (e.g., Malamud and Rostek (2017), Babus and Kondor (2018) and Lambert et al. (2018)), it is hard to analyze dynamic models with heterogeneous market participants. The main difficulty is due to difficult filtering problems where market participants try to infer the source of the observed order flows. A notable exception is the model developed by Sannikov and Skrzypacz (2016), where they avoid this problem by assuming that individual order flows are observable. It is usually simple to analyze the implications of observability of order flows in markets with homogeneous traders. In models such as that of Kyle (1985), non-anonymous order flows result in perfectly revealing prices, and trade breaks down. On the other hand, nothing will change in settings such as that of Antill and Duffie (2019), where inventory shocks drive trade. Anonymity of order flows has been studied before (e.g., Seppi (1990), Admati and Pfleiderer (1991) and Madhavan (1996)). Previous research on the topic distinguishes between informed and uninformed traders, and informed traders benefit from anonymity because they can pool their orders with uninformed traders. The model in this chapter differs in two dimensions. All traders may be symmetrically informed about the future price through their inventory shocks. The second is that traders who prefer an anonymous market may still reveal their identity due to the equilibrium behavior of other traders. Ollar et al. (2017)

analyze how anonymity affects allocations in a model that is similar to the one in this chapter. They study how exogenously specified anonymity affects the ability to attain the first-best allocations with multiple trading rounds.

The contribution of this chapter is to understand incentives to stay anonymous in financial markets. It shows the importance of considering the equilibrium behavior of other traders when a trader decides if he or she wants to reveal their identity. It also provides a game-theoretical justification for the crucial assumption of observable order flows in Sannikov and Skrzypacz (2016). The model in this chapter is similar in spirit with similar implications. The difference is that the two-period model is simpler than the dynamic model, and it is possible to solve the model with both observable and unobservable order flows. Order flows are observable in the equilibrium of the simple model, and it supports the assumption by Sannikov and Skrzypacz (2016). The model in this chapter has the opposite conclusion as the results in Yang and Zhu (2017), where traders may add noise to their order flow to make it less informative. Their results differ because they study a setting where traders who are informed about fundamentals want to hide this information from other traders. The mechanism in my chapter is that some traders want to reveal their identity because they have lower incentives for bid shading, and hence they obtain lower price impacts than if they are anonymous.

3.3 Model setup

I propose a simple model with heterogeneous strategic traders who trade to reduce their inventory costs. The setup of the model is as follows:

Market structure: Trade of a single divisible asset takes place at times $t = 1, 2$. All strategic traders submit market orders as in Kyle (1985) at both times. A competitive risk-neutral market maker determines the price P_t of the asset at time t . The market maker breaks even in expectation and can only hold inventory at time $t = 1$. One can think of the market maker as a group of competitive high-frequency traders who absorb inventory imbalances, but try to keep their total positions small.

Information structure: Total inventory, \bar{X} , is revealed at time $t = 2$. At time $t = 1$, every trader observes his inventory. A trader can truthfully reveal his order flow, but not his inventory. The market maker observes total order flow and the order flow of every non-anonymous trader.

Strategic traders: There are N strategic traders where trader i has inventory cost b_i with $0 < b_N \leq \dots \leq b_1 \leq \infty$. Traders receive inventory shocks with distribution $X_i \sim N(0, \sigma_i^2)$ at time $t = 0$. After submission of the market order, a trader can reveal his order flow. All traders decide whether to reveal their order flow information or stay anonymous simultaneously¹². Trader i maximizes his expected profits net of quadratic inventory costs¹³: That is, trader i maximizes $E[U_i | X_i]$ where

$$U_i = -\frac{b_i(X_{i1})^2}{2} - P_1 Y_{i1} - \frac{b_i(X_{i2})^2}{2} - P_2 Y_{i2}.$$

Here, $X_{i1} = X_i + Y_{i1}$ and $X_{i2} = X_{i1} + Y_{i2}$ are trader i 's inventories at times $t = 1, 2$, and Y_{i1}, Y_{i2} are the market orders. As in common in the literature, I focus on equilibria with linear demand and

¹²This assumption can be changed to any arbitrary order of revealing order flow information without affecting the equilibrium outcomes.

¹³Preferences of this type are popular in the market microstructure literature where the quadratic term is intended to capture elements such as risk-aversion, leverage constraints, or other frictions that makes it costly to sit on inventories. The structure can be endogenized with if the asset pays a risky, normally distributed dividend and investors have exponential utility towards the risky dividend, but are risk-averse towards the asset price.

price functions. Namely, I look for equilibria in which Y_{i1} is a linear function of X_i , Y_{i2} is a linear function of X_{i1} and \bar{X} , and P_1 is a linear function of the revealed order flows and total order flow. Total inventories are observable in period 2 and the price is set such that the market clears. In addition to the strategic traders, there are noise traders who submit orders with distribution $N(0, \sigma_n^2)$ with $\sigma_n^2 \geq 0$. I endogenous the noise in Appendix C.1.2 by making it costly for traders to reveal their identity.

Definition 3. *A set of linear demand functions Y_{i1} and Y_{i2} , linear pricing rules P_1, P_2 , and disclosure policy $z_i \in \{\text{reveal order flow, do not reveal order flow}\}$ is a Linear Bayesian Nash equilibrium if*

- (i) *For every trader i , the expected payoff from the strategy $\{Y_{i1}, Y_{i2}, z_i\}$ is at least as high as any other strategy $\{Y'_{i1}, Y'_{i2}, z'_i\}$ conditional on inventories X_i , the pricing rules P_1 and P_2 , and the strategies of the other traders $\{Y_{j1}, Y_{j2}, z_j\}_{j \neq i}$.*
- (ii) *The market maker sets prices to (1) break even on average and (2) clear the market at time 2 conditional on his information and the strategies of traders.*

3.4 Model solution

3.4.1 Equilibrium with exogenous anonymity

The first step is to solve the model for the optimal behavior when traders take the anonymity decisions as exogenously specified. The linear equilibrium is given by:

Lemma 8. *The unique linear equilibrium when anonymity is exogenous is characterized by:*

- (i) *The equilibrium price at time $t = 2$ is $P_2 = -\bar{b}\bar{X}$ where $\bar{b} = \frac{1}{\sum_i \frac{1}{b_i}}$.*
- (ii) *The order flow of trader i at time $t = 1$ is $Y_{i1} = -\frac{\bar{b}+b_i}{b_i+2\lambda_i} X_i$ where λ_i is the trader's price impact. The expected utility of trader i decreases in λ_i .*
- (iii) *If trader i reveals his order flow, his price impact is $\lambda_i = \frac{\bar{b}b_i}{b_i-\bar{b}}$ with $\frac{\partial \lambda_i}{\partial b_i} < 0$. The price impact of traders who trade anonymously, λ , is a convex combination of the price impacts they would have obtained by revealing their order flow. The price impacts are bounded from below by $\lambda_i \geq \bar{b}$ and $\lambda \geq \bar{b}$.*

Proof: see Appendix C.2.1.

The first part of the Lemma is the behavior at time $t = 2$. There is no asymmetric information about \bar{X} in this period, and the price is the market clearing price. The impact of anonymity is at time $t = 1$. Traders submit market orders that depend on their price impact. If their price impact is higher, then they will submit smaller orders relative to their inventories. Given the bounds of the price impacts, it is clear that $|Y_{i1}| < |X_i|$, which causes an inefficiency because traders sit on inventories that they could have offloaded. All traders prefer that their price impact is lower, and their decisions to trade anonymously or not depends on whether they anticipate a lower price impact by doing so. The last part of Lemma 8 is the important part in the subsequent section. The price impact for anonymous traders is a convex combination of the price impacts they would have obtained if they revealed their

identity. As a result, at least one of those traders would receive a lower price impact by revealing his or her identity with one exception. The only exception is when all anonymous traders are homogeneous, and in this case they have the same price impact regardless of whether they decide to stay anonymous or not.

The proof of Lemma 8 does not contain any noise traders. This is without loss of generality and we can add noise traders if one or more traders have inventory cost parameters $b_i = \infty$, and trade anonymously. In that case, σ_n^2 is the sum of the variance of these traders' inventory shock variances.

3.4.2 Equilibrium choice of anonymity

Proposition 8. *For any amount of noise trading, σ_n^2 , all equilibrium choices of trading anonymously are equivalent to an equilibrium where:*

(i) *All traders with $b_i > b_j$ reveal their identity.*

(ii) *All traders with $b_i \leq b_j$ trade anonymously.*

(iii) *The threshold is unique and satisfies:*

- $j \leq N$ for all σ_n^2 .
- $j = N$ if $\sigma_n^2 = 0$.
- $j = 1$ if $\sigma_n^2 > 0$ and $b_i = b \forall i$.

(iv) *The price impact for an anonymous trader is $\lambda \in [\lambda_{j-1}, \lambda_j]$.*

Proof: see Appendix C.2.2.

There is a unique threshold such that all traders who have a higher cost of holding inventory choose to reveal their identity. The intuition is as follows: traders with a high cost of holding inventory have lower incentives for bid shading, and they therefore obtain low price impacts if they reveal their identity. Traders who has low costs of holding inventory will have a higher price impact if they reveal their identity, and they wish to pool with other traders who trade anonymously, but only if there are some noise traders.

If there is no noise, then the equilibrium structure is simple, all traders except the one(s) with the lowest cost of holding inventory strictly prefer to reveal their identity. This is not because all traders prefer a completely anonymous market, but because all traders who reveal their identity prefer it over the partially anonymous market that is their alternative. The trader(s) with the lowest cost of holding inventories is indifferent between trading anonymously and not when there is no noise. Given that there is only one trader or multiple homogeneous traders who stay anonymous, their price impact and behavior will be exactly the same as if they revealed their identity.

One element that is absent in the proposition is σ_i^2 . It does affect the equilibrium indirectly through the value of the threshold type b_j , but not the equilibrium structure. What drives the final price is the total inventories, and traders who usually have large inventory shocks will be the most informed about

the future price developments. The previous literature has focused on the difference of informed and uninformed traders in questions about anonymity. The results in this chapter suggests that this is no longer the relevant dimension for markets where the main friction is inventory costs. One example is a large, passive fund that is both well informed (large σ_i^2) and prefers to not deviate from the index they follow (large b_i). The previous literature would have suggested that they prefer to stay anonymous based on their information, whereas the results in Proposition 8 suggests that they prefer to reveal their identity because they have low incentives for bid shading.

The results of this chapter is in line with the empirical literature on anonymity and trading behavior. If there is little noise, then one would suggest that most traders prefer to reveal their identity, which is exactly what traders did when they had the option to choose (Comerton-Forde et al. (2011)), and the ones who stay anonymous has a higher price impact than the ones who do not. If a stock exchange changes from a non-anonymous to an anonymous structure, then we would expect the traders who have stronger incentives to split an order into multiple smaller orders, can do so with a smaller price impact, whereas the remaining traders get a higher price impact. This is in line with the results from Meling (2019) who finds that the introduction of anonymity increased the amount of trading by institutional investors (low b_i), but had a smaller effect on retail investors (high b_i). Furthermore, the change to anonymity received mixed support from the constituents of the exchange, and the policy was eventually reversed. This behavior suggests that some investors gain whereas others loose from anonymity.

3.4.3 Anonymity and welfare with two types

There are two elements that affect welfare for individual traders, and one that affect total welfare. Individual traders care about their inventory costs and the price at which they trade. The total welfare only depends on inventory costs as the price is just a transfer that sums to 0. To obtain theoretical results, I put additional structure on the problem with two types of traders, patient traders ($b_i = b < \infty$) and impatient traders ($b_i = \infty$). One can think of patient traders as sophisticated investors and impatient traders as small retail investors.

Proposition 9. *Compared to a non-anonymous market, a fully anonymous market results in:*

- (i) *Patient traders have higher welfare.*
- (ii) *Impatient traders have lower welfare.*
- (iii) *Total welfare is higher.*

Proof: see Appendix C.2.3.

Patient traders have a lower price impact when the market is anonymous compared to a non-anonymous market, and vice versa for impatient traders. Traders prefer to have a lower price impact, which explains the welfare results of each of the types. Total welfare depends on total inventory costs. Patient traders submit larger orders when their price impact is lower, which reduces their inventory costs. The behavior of impatient traders is independent of their price impact, they always submit orders that have exactly the same size as their inventories, and anonymity does not affect their

inventory costs. The total effect is always a reduction in total inventory costs in this example, which increases total welfare¹⁴.

The result also provides an explanation for varying practices when it comes to anonymity. Even if anonymity increases total welfare as in Proposition 9, it distributes these gains unevenly. Regulators may put more weight on some traders' welfare when they make their decisions, and we will see different markets that differ in the choice of anonymity.

3.5 Conclusion

I show that if anonymous trading is a choice, then anonymity is only the equilibrium outcome if markets are sufficiently noisy and traders sufficiently homogeneous. Any equilibrium without noisy order flow is equivalent to an equilibrium where all traders reveal their order flow. Traders reveal their identity not because all of them prefer the non-anonymous market, but because they anticipate that some traders do prefer to reveal their identity. In equilibrium, this will induce some traders, who would have preferred full anonymity, to reveal their identity rather than being anonymous in a semi-anonymous market.

My results suggest that the anonymous market is more efficient than the non-anonymous market in terms of total welfare, but the gains are unevenly distributed. Patient traders benefit from a lower price impact at the expense of impatient traders, and regulators may put more weight on either type of trader's welfare. This explains why the amount of transparency about trader identities differ between different trading venues, and that some exchanges, such as Oslo Stock Exchange, have introduced and later removed post-trade anonymity.

While my model is simple, the central intuition will likely extend to a richer set of models, such as the dynamic model of Sannikov and Skrzypacz (2016).

¹⁴Simulations suggest that this is also the case for general choices of b_i . See Appendix C.1.1.

A Appendix - Chapter I

A.1 Extensions

A.1.1 Endogenous noisy supply

One way to make the noisy supply endogenous is to have some traders who face idiosyncratic risk, $\tilde{z}\varepsilon_i$, where $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2)$ with $Cov(\varepsilon, \varepsilon_i) = \rho\sigma_{\varepsilon_i}\sigma_e$. A fraction $\omega \in (0, 1)$ of the informed traders face this risk. The demand of the asset for a trader who face the idiosyncratic risk solves

$$\sup_{X_i} E[W|s, P, \tilde{z}] - \frac{a}{2} Var(W|s, P, \tilde{z})$$

where $W = X_i(I(s + \varepsilon) + \varepsilon_0 - P) + \tilde{z}\varepsilon_i$. The solution to the maximization problem is

$$X_i = \frac{I(s + \tilde{z}\rho\sigma_e\sigma_{\varepsilon_i}) - P}{a(I^2\sigma_e^2 + \sigma_0^2)}.$$

The demand of the traders who face idiosyncratic risk is

$$\omega\lambda AX_i = \omega\lambda A \frac{Is - P}{a(I^2\sigma_e^2 + \sigma_0^2)} + \omega\lambda A \frac{\rho\sigma_{\varepsilon_i}}{a\sigma_e} \hat{f}(I)\tilde{z}.$$

Thus, we obtain noise trading of the form that is assumed in the paper. If $\sigma_0^2 = 0$, then we can also endogenize it with uninformed traders who face idiosyncratic risk towards s and/or ε in exactly the same way.

A.1.2 Noise trader demand times price

The traditional way of modelling noise traders is to specify their demand in the number of assets, which is also what I do in this paper. If the noise comes from retail investors, then it is perhaps more realistic to think of their random demand to be in terms of the price they pay, that is $P(s_u)z$. The following parametric example shows that the distribution of $P(s_u)z$ may depend less on the real investment behavior than the demand in terms of number of assets. To simplify the algebra, let $\sigma_0^2 = 0$,

$\bar{s}_0 = 0$, and $I_0 = 0$. Then the security price is

$$P(s_u) = E[s|s_u]I - \frac{c}{2}I^2 - \frac{a_u((1-c_1)\sigma_s^2 + \sigma_e^2)}{a_u((1-c_1)\sigma_s^2 + \sigma_e^2) + a_i\sigma_e^2}(a_i\sigma_e^2I^2\bar{z} - (1-c_1)Is_u).$$

This gives the following expression for $P(s_u)z$:

$$P(s_u)z = \left(E[s|s_u] + \frac{a_u((1-c_1)\sigma_s^2 + \sigma_e^2)}{a_u((1-c_1)\sigma_s^2 + \sigma_e^2) + a_i\sigma_e^2}(1-c_1)s_u - \left(\frac{c}{2} + \frac{a_u a_i((1-c_1)\sigma_s^2 + \sigma_e^2)\sigma_e^2\bar{z}}{a_u((1-c_1)\sigma_s^2 + \sigma_e^2) + a_i\sigma_e^2} \right) I \right) z.$$

The terms in the first line are independent of real investment behavior whereas the second line is affected by real investment. Feedback effects affect the distribution of what the noise traders demand in the parametric example, but only through the risk premium and the adjustment costs. If noise traders have a fixed budget they want to spend on the stock, then it is more natural to let $P(s_u)z$ be independent of I , rather than z itself. While the specification of z in this paper does not get exactly that result, it is perhaps a better approximation than one where z itself is normally distributed.

A.1.3 Imperfectly correlated signal

The main model has perfectly correlated signals for informed traders. The solution technique also works perfectly well for imperfectly correlated signals à la Hellwig (1980). Let the payoff be

$$V = sI - \frac{c}{2}(I - I_0)^2$$

where every trader observes a signal $s_i = s + \epsilon_i$. The signal noise, ϵ_i is independent between traders. Noise traders demand a random exposure to s , $z = \frac{\bar{z}}{I}$. All shocks are normal with $s \sim N(\bar{s}, \sigma_s^2)$, $\epsilon_i \sim N(0, \sigma_e^2)$ and $\bar{z} \sim N(0, \sigma_z^2)$. Furthermore, I conjecture that there is an equilibrium where the price is an increasing function of $s_u = s + bz$ for some b . As above, I assume that the preferences or information of the manager ensure that real investment is a function of price and hence also a function of s_u .

The distribution of V for trader i conditional on s_i and s_u is

$$V_{|s_i, s_u} \sim N\left(E[s|s_i, s_u]I - \frac{c}{2}(I - I_0)^2, I^2 \text{Var}(s|s_i, s_u)\right).$$

Given the multivariate normal structure, it is straightforward to show that

$$E[s|s_i, s_u] = (1 - c_1 - c_2)\bar{s} + c_1 s_i + c_2 s_u$$

for $c_1 = \frac{b^2\sigma_s^2\sigma_z^2}{\sigma_e^2\sigma_s^2 + b^2(\sigma_e^2 + \sigma_s^2)\sigma_z^2}$ and $c_2 = \frac{\sigma_e^2\sigma_s^2}{\sigma_e^2\sigma_s^2 + b^2(\sigma_e^2 + \sigma_s^2)\sigma_z^2}$. The conditional variance is

$$\text{Var}(s|s_i, s_u) = \frac{b^2\sigma_e^2\sigma_s^2\sigma_z^2}{\sigma_e^2\sigma_s^2 + b^2(\sigma_e^2 + \sigma_s^2)\sigma_z^2}.$$

As in Lemma 2 and Lemma 3 we have

$$X_i = \frac{E[V|s_i, s_u] - P}{aI^2 \text{Var}(s|s_i, s_u)}.$$

This gives the market clearing condition

$$\begin{aligned} \int_0^A X_i di &= \bar{z} + \frac{\bar{z}}{I} \\ \int_0^A \frac{((1-c_1-c_2)\bar{s} + c_1 s_i + c_2 s_u)I - \frac{c}{2}(I-I_0)^2 - P}{aI^2 \text{Var}(s|s_i, s_u)} di &= \bar{z} + \frac{\bar{z}I}{I^2} \\ A \frac{((1-c_1-c_2)\bar{s} + c_2 s_u)I - \frac{c}{2}(I-I_0)^2 - P}{aI^2 \text{Var}(s|s_i, s_u)} + \int_0^A \frac{c_1 s_i I}{aI^2 \text{Var}(s|s_i, s_u)} di &= \bar{z} + \frac{\bar{z}I}{I^2} \\ A \frac{((1-c_1-c_2)\bar{s} + c_1 s + c_2 s_u - \frac{a}{A} \text{Var}(s|s_i, s_u)\bar{z})I - \frac{c}{2}(I-I_0)^2 - P}{aI^2 \text{Var}(s|s_i, s_u)} &= \bar{z}. \end{aligned}$$

Solving this for P gives

$$P(s_u, I) = ((1-c_1-c_2)\bar{s} + c_1 s + c_2 s_u - \frac{a}{A} \text{Var}(s|s_i, s_u)\bar{z})I - \frac{c}{2}(I-I_0)^2 - \frac{a}{A} I^2 \text{Var}(s|s_i, s_u)\bar{z}.$$

We can rewrite $c_1 s - \frac{a}{A} \text{Var}(s|s_i, s_u)\bar{z}$ as

$$c_1 \left(s + \frac{-\frac{a}{A} \text{Var}(s|s_i, s_u)\bar{z}}{c_1} \right).$$

We get a fixed point equation for b :

$$\frac{-\frac{a}{A} \text{Var}(s|s_i, s_u)}{(c_1)} = b$$

Plugging in values and simplifying gives

$$-\frac{a\sigma_e^2}{A} = b.$$

The unique equilibrium has $b = -\frac{a\sigma_e^2}{A}$. This gives an equilibrium price

$$P(s_u, I) = ((1-c_1-c_2)\bar{s} + (c_1 + c_2)s_u)I - \frac{c}{2}(I-I_0)^2 - \frac{a}{A} I^2 \text{Var}(s|s_i, s_u)\bar{z}.$$

For a specific investment function, $I(s_u)$, we need to verify that the price, $P(s_u, I(s_u))$, is indeed increasing in s_u .

A.1.4 General specification of risk

Suppose that the final payoff takes the form

$$V = Is + f(I)\varepsilon + \varepsilon_0 - C(I)$$

where $f(I)$ and $C(I)$ are functions of I . All random variables are normally distributed with $s \sim N(\bar{s}, \sigma_s^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, and $\varepsilon_0 \sim N(\bar{\varepsilon}_0, \sigma_0^2)$. An informed investor observes s and the price whereas an uninformed investor only observes the price. The market clearing condition is

$$\lambda AX_i + (1-\lambda)AX_u = \bar{z} + \hat{f}(I)\bar{z}$$

where $\hat{f}(I) = \frac{I}{f(I)^2 + \frac{\sigma_0^2}{\sigma_e^2}}$ and $\tilde{z} \sim N(0, \sigma_z^2)$. This specification ensures that the equilibrium signal-to-noise ratio is constant. We can see that there is a one-to-one map between the specification of risk and noise trading. This specification can be endogenized in exactly the same way as in Section A.1.1. We obtain the standard specification of normally distributed demand if $f(I) = \sqrt{I}$ and $\sigma_0^2 = 0$. The solution technique requires that the manager has an optimization problem where the optimal investment is a function of s_u , $I(s_u)$. One example is a manager who maximizes $E[V|s_u]$.

Given the above specification, we can use the same derivation as in Proposition 1 and obtain an equilibrium price

$$P(s_u; I) = \underbrace{\bar{s}_0 + E[s|s_u]I - C(I)}_{\text{Expected value}} - \underbrace{\frac{d_1(I)d_2(I)}{d_1(I) + d_2(I)}\tilde{z} + (1 - c_1)\frac{d_1(I)}{d_1(I) + d_2(I)}Is_u}_{\text{Risk premium}}$$

where

$$\begin{aligned} d_1(I) &= a_u(I^2(1 - c_1)\sigma_s^2 + f(I)^2\sigma_e^2 + \sigma_0^2) \\ d_2(I) &= a_i(f(I)^2\sigma_e^2 + \sigma_0^2). \end{aligned}$$

The signal s_u is given by $s_u = s - \frac{a\sigma_e^2}{\lambda A}\tilde{z}$. The condition for an equilibrium in the information market is exactly the same as in Proposition 2. I did not use the specification of $f(I)$ or $C(I)$ to derive the equilibrium in the information market and the proof goes through without any adjustments.

The requirement of Assumption 1 may not be satisfied for all specifications. It is always satisfied if the manager who maximizes the price but may fail for a manager who maximizes $E[V|s_u]$. The reason is that the manager's risk preferences may not be aligned with those of the market participants so that he or she increases investment too fast relative to what the market participants would have wanted.

Example: Information acquisition equilibrium with $f(I) = I^\alpha$.

Similar results as those of Corollary 4 are also present when $\sigma_0^2 = 0$ and $f(I) = I^\alpha$. It is straightforward to show that

$$1 - e^{ak} \sqrt{\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}}$$

is strictly concave for $\alpha \in [\frac{1}{2}, 1)$ and strictly convex for $\alpha \in (1, 2]$. When the investment function is symmetric, we can use Jensen's inequality and obtain the same economic implications as in Corollary 4. In this setting, feedback effects lower the incentives to become informed if $\alpha \in [\frac{1}{2}, 1)$ and vice versa if $\alpha \in (1, 2]$.

A.2 Proofs

A.2.1 Proof of Lemma 1

Define W_{j1} as the wealth after one trading round and W_j as the final wealth. Furthermore, let $U_j(W_{j1})$ be the value function for trader j in the second trading round with wealth W_{j1} . Define X_{j1} as position

of trader j in the first trading round. The wealth after the first trading round is given by

$$W_{j1} = X_{j1}(P_2 - P).$$

The optimization problem is given by

$$\sup_{X_{ij}} \beta E[-e^{-aX_{j1}(V-P)} | F_{j1}] + (1 - \beta) E[U_j(W_{j1}) | F_{j1}]$$

The first order condition is

$$\beta \frac{\partial E[-e^{-aX_{j1}(V-P)} | F_{j1}]}{\partial X_{j1}} + (1 - \beta) E[U'_j(W_{j1})(P_2 - P) | F_{j1}] = 0$$

When $P_2 = P$, then this simplifies to

$$\frac{\partial E[-e^{-aX_{j1}(V-P)} | F_{j1}]}{\partial X_{j1}} = 0,$$

and investors behave as if they can only trade in the first trading round.

A.2.2 Proof of Lemma 2

Proof. The informed investor maximize $E[-e^{-aW} | s, P]$ where $W = X_i(V - P)$ with respect to X_i . Conditional on observing P and s , V is normally distributed and we can write the expected utility as

$$E[-e^{-aW} | s, P] = -e^{E[-aW | s, P] + \frac{1}{2} \text{Var}[-aW | s, P]}.$$

This is equivalent to maximizing

$$E[aW | s, P] - \frac{1}{2} \text{Var}[aW | s, P]. \quad (\text{A.1})$$

The expected value can be rewritten as

$$E[aW | s, P] = aX_i(I(P)s + \bar{s}_0 - \frac{c}{2}(I(P) - I_0)^2 - P)$$

and the variance as

$$\text{Var}[aW | s, P] = a^2 X_i^2 (I(P)^2 \sigma_e^2 + \sigma_0^2).$$

If we plug this into the (A.1), we get

$$aX_i(I(P)s + \bar{s}_0 - \frac{c}{2}(I(P) - I_0)^2 - P) - \frac{1}{2} a^2 X_i^2 (I(P)^2 \sigma_e^2 + \sigma_0^2).$$

The FOC wrt X_i gives

$$X_i = \frac{I(P)s - \frac{c}{2}(I(P) - I_0)^2 - P}{a(I(P)^2\sigma_e^2 + \sigma_0^2)} = \hat{f}(I) \frac{s}{a\sigma_e^2} - \hat{f}(I) \frac{P + \frac{c}{2}(I(P) - I_0)^2 - \bar{s}_0}{aI\sigma_e^2}.$$

□

A.2.3 Proof of Lemma 3

Proof. As in the proof above, the uninformed investors will maximize $E[-e^{-aW}|s_u, P]$. The only difference is the information they condition their decision on. W is still normally distributed conditional on their information set, and they also maximize exponential utility, which can be simplified to maximizing

$$aX_u(E[V|s_u, P] - P) - a^2 X_u^2 \text{Var}(V|s_u, P).$$

The first-order condition is

$$X_u = \frac{E[V|s_u, P] - P}{a \text{Var}[V|s_u, P]}.$$

The expected value of V is given by

$$\begin{aligned} E[V|s_u, P] &= I(P)E[s|s_u, P] + \sigma_0^2 - \frac{c}{2}(I(P) - I_0)^2 \\ &= I(P)\left(\frac{\text{Cov}(s, s_u)}{\text{Var}(s_u)}(s_u - E[s_u])\right) + \sigma_0^2 - \frac{c}{2}(I(P) - I_0)^2 \\ &= I(P)\left(\frac{\sigma_s^2}{\sigma_s^2 + c_0^2\sigma_z^2}(s_u - 0) + \sigma_0^2 - \frac{c}{2}(I(P) - I_0)^2\right) \\ &= I(P)c_1 s_u + \sigma_0^2 - \frac{c}{2}(I(P) - I_0)^2. \end{aligned}$$

The variance of V is given by

$$\begin{aligned} \text{Var}(V|s_u, P) &= I(P)^2(\text{Var}(s|s_u, P) + \sigma_e^2) + \sigma_0^2 \\ &= I(P)^2\left(\left(1 - \frac{\sigma_s^4}{\sigma_s^2(\sigma_s^2 + c_0^2\sigma_e^2)}\right)\sigma_s^2 + \sigma_e^2\right) + \sigma_0^2 \\ &= I(P)^2((1 - c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2. \end{aligned}$$

This gives the uninformed demand

$$X_u = \frac{I(P)c_1 s_u - \frac{c}{2}(I(P) - I_0)^2 - P + \bar{s}_0}{a(I(P)^2((1 - c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2)}.$$

□

A.2.4 Proof of Proposition 1

Equilibrium price

The market clearing condition, (1.2), is linear in P and there has to be a unique solution. Solving the linear equation gives (1.3) where $E[s|s_u] = c_1 s_u$.

Assumption 1 satisfied

Let $I_1 = I^*(s_{u1})$, $I_2 = I^*(s_{u2})$, and $s_{u1} < s_{u2}$. From

$$I^*(s_u) = \operatorname{argmax}_I P(s_u; I)$$

we know that $P(s_{u2}; I_2) \geq P(s_{u2}; I_1)$. Furthermore, we have

$$\begin{aligned} P(s_{u2}; I_1) - P(s_{u1}; I_1) &= (E[s|s_{u2}] - E[s|s_{u1}])I_1 + (1 - c_1) \frac{d_1(I_1)}{d_1(I_1) + d_2(I_1)} I_1 (s_{u2} - s_{u1}) \\ &= I_1 \left(c_1 + (1 - c_1) \frac{d_1(I_1)}{d_1(I_1) + d_1(I_2)} \right) (s_{u2} - s_{u1}) > 0. \end{aligned}$$

Hence, we have $P(s_{u2}; I^*(s_{u2})) > P(s_{u1}; I^*(s_{u1}))$ and Assumption 1 is satisfied.

Investment as a function of s_u

Suppose that the equilibrium real investment is a deterministic function of s_u , $I(s_u)$. Then we need to specify the beliefs of the informed and uninformed investors if the manager deviates from the equilibrium behavior. One alternative is that off-equilibrium beliefs are unaffected by the deviation. That is, an informed trader's belief is $s + \varepsilon \sim N(s, \sigma_e^2)$ and an uninformed trader's belief is $s + \varepsilon \sim N(c_1 s_u, (1 - c_1)\sigma_s^2 + \sigma_e^2)$. Conditional on these off-equilibrium beliefs, the price in the second trading round is given by (1.3), which is a function of s_u , I , and the parameters of the model. However, it is independent of other information in \mathcal{F}_m . Hence, the optimal real investment for a manager is a function of s_u but is independent of other information in \mathcal{F}_m . Therefore there exist a pooling equilibrium where the manager disregards his or her private information.

A.2.5 Proof of Proposition 2

Conditional expected utility for informed trader

This proof follows that of Grossman and Stiglitz (1980). Define $Q = V - P$. The optimal portfolio choice is

$$X_i = \frac{E[Q|s, s_u]}{a \text{Var}(V|s, s_u)}.$$

We need to calculate $E[U_i|s, s_u] = E[-e^{-aW_i}]$. The conditional distribution of $-aW_i = ak - \frac{E[Q|s, s_u]Q}{\text{Var}(V|s, s_u)}$ is normally distributed with

$$E[-aW_i|s, s_u] = ak - \frac{E[Q|s, s_u]^2}{\text{Var}(V|s, s_u)}$$

and

$$\text{Var}(-aW_i|s, s_u) = \frac{E[Q|s, s_u]^2}{\text{Var}(V|s, s_u)}.$$

As a result, we have

$$E[U_i|s, s_u] = -e^{ak} e^{-\frac{E[Q|s, s_u]^2}{2\text{Var}(V|s, s_u)}}.$$

Next, define

$$h = \text{Var}(E[V|s, s_u]|s_u) = I^2(s_u) \text{Var}(s|s_u)$$

and

$$Z = \frac{E[Q|s, s_u]}{\sqrt{h}}.$$

We need to calculate

$$E[U_i|s_u] = -e^{ak} E[e^{-tZ^2} |s_u]$$

where $t = \frac{h}{2\text{Var}(V|s, s_u)}$. Z is normally distributed with

$$E[Z|s_u] = \frac{E[Q|s_u]}{\sqrt{h}}$$

due to the law of iterated expectations and unit variance. Hence, Z^2 has a non-central χ^2 distribution with moment generating function

$$E[e^{-tZ^2} |s_u] = \frac{1}{\sqrt{1+2t}} e^{-\frac{E[Z|s_u]^2 t}{1+2t}}.$$

Plugging in $E[Z|s_u] = \frac{E[Q|s_u]}{\sqrt{h}}$ and $t = \frac{h}{2\text{Var}(V|s, s_u)}$ gives

$$E[U_i|s_u] = -e^{ak} \sqrt{\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}} e^{-\frac{E[Q|s_u]^2}{2\text{Var}(V|s_u)}} \quad (\text{A.2})$$

Conditional expected utility for uninformed trader

Following similar steps as above, we can easily show that

$$E[U_u|s_u] = -e^{-\frac{E[Q|s_u]^2}{2\text{Var}(V|s_u)}}. \quad (\text{A.3})$$

Expected utility conditional on $I = 0$

If $0 \in \mathcal{I}$, then there exist a threshold \bar{s}_u such that $I = 0$ if $s_u \leq \bar{s}_u$. In this case, the security market equilibrium does not reveal s_u . We can however keep all the above calculations such that

$$E[U_i | s, s_u \leq \bar{s}_u] = -e^{ak} e^{-\frac{E[Q|s_u \leq \bar{s}_u]^2}{2\text{Var}(V|s_u \leq \bar{s}_u)}}$$

and

$$E[U_u | s_u \leq \bar{s}_u] = e^{-\frac{E[Q|s_u \leq \bar{s}_u]^2}{2\text{Var}(V|s_u \leq \bar{s}_u)}}.$$

$E[Q|s_u \leq \bar{s}_u]$ and $\text{Var}(V|s_u \leq \bar{s}_u)$ are constants. Hence, the revelation of s_u for $s_u \leq \bar{s}_u$ will not change anything and

$$E[U_j | s_u \leq \bar{s}_u] = E[U_j | s_u]$$

for $j = i, u$ when $s_u \leq \bar{s}_u$. We can therefore use (A.2) and (A.3) even when $I = 0$ to shorten notation.

Information equilibrium

In an interior equilibrium, we need $E[U_i] - E[U_u] = 0$. Using (A.2) and (A.3) we get the condition

$$E[U_i] - E[U_u] = E \left[\left(1 - e^{ak} \sqrt{\frac{\text{Var}(V|s, s_u)}{\text{Var}(V|s_u)}} \right) e^{-\frac{(E[V|s_u]) - P(s_u))^2}{2\text{Var}(V|s_u)}} \right].$$

A.2.6 Proof of Lemma 4

The function $g(I) = 1 - e^{ak} \sqrt{\frac{I^2 \sigma_e^2 + \sigma_0^2}{I^2((1-c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2}}$ has the following first and second derivatives:

$$g'(I) = \frac{e^{ak} I (1-c_1) \sigma_s^2 \sigma_0^2}{\sqrt{\frac{I^2 \sigma_e^2 + \sigma_0^2}{I^2((1-c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2}} (I^2((1-c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2)^2}$$

and

$$g''(I) = \hat{g}(I) (\sigma_0^2 - 2\sigma_0^2((1-c_1)\sigma_s^2 + \sigma_e^2)I^2 - 3\sigma_e^2((1-c_1)\sigma_s^2 + \sigma_e^2)I^4) \quad (\text{A.4})$$

where $\hat{g}(I) = \frac{e^{ak}(1-c_1)\sigma_s^2\sigma_0^2}{\left(\sqrt{\frac{I^2\sigma_e^2 + \sigma_0^2}{I^2((1-c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2}}\right)^3 (I^2((1-c_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2)^4} \geq 0$.

If $\sigma_0^2 = 0$, then it is straightforward to see that $g(I)$ is a constant. If $\sigma_0^2 > 0$, then $g'(I) > 0$ for all $I > 0$. For the second derivative we have

$$\text{Sign}[g''(I)] = \text{Sign}[\sigma_0^2 - 2\sigma_0^2((1-c_1)\sigma_s^2 + \sigma_e^2)I^2 - 3\sigma_e^2((1-c_1)\sigma_s^2 + \sigma_e^2)I^4].$$

It is straightforward to see that there exist a unique positive threshold $I = \bar{I}$ such that

$$\sigma_0^2 - 2\sigma_0^2((1-c_1)\sigma_s^2 + \sigma_e^2)\bar{I}^2 - 3\sigma_e^2((1-c_1)\sigma_s^2 + \sigma_e^2)\bar{I}^4 = 0. \quad (\text{A.5})$$

The second derivative is positive for $I < \bar{I}$ and negative for $I > \bar{I}$.

A.2.7 Proof of Corollary 2

When $\sigma_0^2 = 0$, then $g(I)$ is a constant independent of I . Hence, we have

$$\Delta_U(c_1) = E[g(I)e^{-\frac{(E[V|s_u]-P(s_u))^2}{2\text{Var}(V|s_u)}}] = g(I)E[e^{-\frac{(E[V|s_u]-P(s_u))^2}{2\text{Var}(V|s_u)}}].$$

We have $\text{Sign}(\Delta_U(c_1)) = \text{Sign}(g(I))$ and all equilibrium properties are the same as in Grossman and Stiglitz (1980). I.e., the equilibrium is unique. An interior equilibrium satisfies

$$0 = 1 - e^{ak} \sqrt{\frac{\sigma_e^2}{(1-c_1)\sigma_s^2 + \sigma_e^2}},$$

which can be rewritten as

$$c_1 = 1 - \frac{\sigma_e^2}{\sigma_s^2} (e^{2ak} - 1).$$

A.2.8 Proof of Corollary 3

Part (1)

If $k < \bar{k}_2$, then $g(I_2; c_1 = 0) > 0$ and there is an interior equilibrium in the information market without feedback effects. Otherwise $\bar{c}_1 = 0$.

For a fixed investment, the function $g(I; c_1)$ is decreasing in c_1 . This is the standard result from Grossman and Stiglitz (1980). The equilibrium condition for an interior equilibrium is

$$E \left[g(I(s_u)) e^{-\frac{(E[V|s_u]-P(s_u))^2}{2\text{Var}(V|s_u)}} \right] = 0.$$

From Lemma 4 we know that $g(I_1; c_1) < g(I_2; c_1)$. Hence, we need $g(I_1; c_1) < 0$ and $g(I_2; c_1) > 0$. An equilibrium without feedback effects has $g(I_2, \bar{c}_1) = 0$ and hence $g(I_2; c_1) > g(I_2, \bar{c}_1)$ and $c_1 < \bar{c}_1$ in an interior equilibrium. If $\lambda = 1$, then $g(I_2; \bar{c}_1) \geq 0$ and there may be an equilibrium where $c_1 = \bar{c}_1$.

If $c_1 = 0$, then $I = I_2$ since there is no new information. As a result, there can not be an equilibrium with $c_1 = 0$ if $\bar{c}_1 > 0$.

Part (2)

First part, if $k > \bar{k}_1$, then $g(I_1) < 0$ for all c_1 . Hence, the equilibrium without feedback effects is unique with no informed investors. If the firm can adjust their investment, then an interior equilibrium requires that $g(I_1; c_1) < 0 < g(I_2; c_1)$ and $c_1 > \bar{c}_1$ in an interior equilibrium. This follows exactly the opposite intuition as in part (1) of this proof.

If $I_1 = 0$ and $c_1 = 0$, then there is no new information and it is an equilibrium to never invest. This is an equilibrium if $k \geq \bar{k}_1 = 0$. If $c_1 > 0$, then $\text{Prob}(I = I_2) > 0$ and an equilibrium requires that $g(I_1; c_1) \leq 0$. It is straightforward to see that $\frac{\partial \Delta_U(c_1)}{\partial k} < 0$. Furthermore, $\Delta_U(c_1; c_1 > 0, k = 0) > 0$ and $\Delta_U(c_1; k \rightarrow \infty) < 0$. Hence, there exist a k with an equilibrium where a fraction $\lambda \in (0, 1]$ of investors

are informed. Due to the continuity of $\Delta_U(c_1; k)$, there is an odd number of equilibria unless the parameters are such that $\sup_{c_1} \Delta_U(c_1; k) = 0$, in which case there may be an even number of equilibria.

A.2.9 Proof of Corollary 4

Part (1)

When $A \rightarrow \infty$, then the maximization problem that maximizes price is equivalent to the one that maximizes $E[V|P]$. If $c_1 = 0$, then the manager does not learn any new information, then $I = I_0$ is optimal since $E[s|P] = 0$ and the manager choose the investment level that minimizes adjustment costs. With the additional assumption that $\bar{c}_1 > 0$ we get $\Delta_U(0) > 0$. Furthermore, when $A \rightarrow \infty$ there exist a λ such that $c_1 \rightarrow 1$.

By continuity of $\Delta_U(c_1)$ it is straightforward to see that if $\Delta_U(\bar{c}_1) > 0$, then there exist a solution where $\Delta_U(c_1) = 0$ with $c_1 > \bar{c}_1$ and vice versa for $\Delta_U(\bar{c}_1) < 0$. The result is obvious for $\Delta_U(\bar{c}_1) = 0$.

Part (2)

We can write $\Delta_U(\bar{c}_1)$ as

$$\begin{aligned} \Delta_U(\bar{c}_1) &= Prob(I = I_0(1 - \epsilon))g(I_0(1 - \epsilon); \bar{c}_1) + Prob(I = I_0(1 + \epsilon))g(I_0(1 + \epsilon); \bar{c}_1) \\ &\quad + (1 - Prob(I = I_0(1 - \epsilon)) - Prob(I = I_0(1 + \epsilon)))g(I_0; \bar{c}_1) \\ &= \frac{Prob(I \neq I_0)}{2}(g(I_0(1 - \epsilon); \bar{c}_1) + g(I_0(1 + \epsilon); \bar{c}_1)). \end{aligned}$$

The equality uses symmetry ($Prob(I = I_0(1 - \epsilon)) = Prob(I = I_0(1 + \epsilon))$) and $g(I_0; \bar{c}_1) = 0$. Next, we can take a third order Taylor expansion of $g(I_0(1 - \epsilon); \bar{c}_1)$ and $g(I_0(1 + \epsilon); \bar{c}_1)$ around $\epsilon = 0$. We have

$$\begin{aligned} g(I_0(1 + \gamma\epsilon); \bar{c}_1) &= g(I_0; \bar{c}_1) + g'(I_0; \bar{c}_1)(I_0(1 + \gamma\epsilon) - I_0) + \frac{1}{2}g''(I_0; \bar{c}_1)(I_0(1 + \gamma\epsilon) - I_0)^2 \\ &\quad + \frac{1}{6}g'''(I_0; \bar{c}_1)(I_0(1 + \gamma\epsilon) - I_0)^3 + O(\epsilon^4) \\ &= g'(I_0; \bar{c}_1)(I_0\gamma\epsilon) + \frac{1}{2}g''(I_0; \bar{c}_1)(I_0\gamma\epsilon)^2 + \frac{1}{6}g'''(I_0; \bar{c}_1)(I_0\gamma\epsilon)^3 + O(\epsilon^4). \end{aligned}$$

The odd terms cancel when we sum the approximation for $\gamma = -1$ and $\gamma = 1$ and we get

$$g(I_0(1 + \epsilon); \bar{c}_1) + g(I_0(1 - \epsilon); \bar{c}_1) = g''(I_0; \bar{c}_1)I_0^2\epsilon^2 + O(\epsilon^4).$$

Next, in an interior equilibrium we need \bar{c}_1 to satisfy

$$1 - e^{ak} \sqrt{\frac{I_0^2\sigma_e^2 + \sigma_0^2}{I_0^2((1 - \bar{c}_1)\sigma_s^2 + \sigma_e^2) + \sigma_0^2}}.$$

This gives

$$\bar{c}_1 = 1 - \frac{(1 - e^{2ak})}{\sigma_s^2} \left(\sigma_e^2 + \frac{\sigma_0^2}{I_0^2} \right).$$

Plugging it into equation (A.4) and simplifying gives the expression from the Corollary.

A.2.10 Proof of Proposition 3

Part (1)

When $A \rightarrow \infty$, then a manager who maximizes the stock price is equivalent to one that maximizes $E[V|s_u]$. Consider c_1^0 and c_1^1 with $c_1^1 > c_1^0$. The optimal investment function when $c_1 = c_1^0$ is $I^0(s_u^0)$ for some function $I^0(\cdot)$.

Next, let the equilibrium efficiency be given by $c_1 = c_1^1$ so that the manager observes s_u^1 . The manager can draw a random normal variable ϵ_1 such that $Var(s_u^0) = Var(s_u^1 + \epsilon_1)$. The distribution of s_u^0 and $s_u^1 + \epsilon_1$ are the same with the same correlations with s . Hence, one feasible real investment strategy for a manager who observes s_u^1 is $I^0(s_u^1 + \epsilon_1)$. This gives the same $E[V]$ as for a manager who observes s_u^0 and behaves optimally by construction. This strategy is optimal for a manager who maximizes $E[V|s_u^0]$ whereas it is not for one who observes s_u^1 . Therefore $E[V]$ is increasing in c_1 .

To show that it is strictly increasing, note that $E[s|s_u^1, s_u^1 + \epsilon_1] = E[s|s_u^1]$ and the optimal real investment is deterministic conditional on s_u^1 . Adding random noise may change real investment, which is suboptimal and reduces real efficiency.

Part (2)

From the third part of this proposition we know that the existence of equilibria and their efficiency is independent of σ_z^2 . The cost of information production for a given equilibrium is however increasing in σ_z^2 . For any c_1 , we have $\lim_{\sigma_z^2 \rightarrow 0} W(c_1; \sigma_z^2) = E[V|c_1]$. From the first part of the proposition we have

$E[V|c_1 > 0] > E[V|c_1 = 0]$. Due to the monotone decrease in $W(c_1)$ with respect to σ_z^2 , there exist a $\bar{\sigma}_z^2$ such that $W(c_1) = W(0)$.

Part (3)

The equilibrium condition when $A \rightarrow \infty$ is $E[g(I)] = 0$ in an interior equilibrium. $g(I)$ does not contain σ_z^2 . Furthermore, the distribution of I does not change if $\frac{\sigma_z}{\lambda A}$ stays constant. Hence, if we change σ_z^2 , we can adjust λ so that the equilibrium investment behavior is unaffected and the equilibrium value of c_1 stays constant. As a result, changes in σ_z^2 does not change the equilibria that exists or their informativeness as long as $\sigma_z^2 > 0$.

If σ_z^2 increases, we need to increase λ so that the equilibrium efficiency stays constant. This increases the cost to produce information and strictly reduces $W(c_1)$.

Part (4)

It is easy to see that $\frac{\partial g(I; k)}{\partial k} < 0$ and therefore we also have $\frac{\partial \Delta_U(c_1; k)}{\partial k} < 0$. As a result, we have $\frac{\partial c_1}{\partial k} < 0$. An increase in k reduces $E[V]$ due to the result in the first part of this proposition.

The cost of information production is λAk . We can rewrite equation (1.1) to get

$$\lambda A = a\sigma_e^2 \frac{\sigma_z}{\sigma_s} \sqrt{\frac{c_1}{1-c_1}}.$$

The effect on information production costs from changing k is given by

$$\frac{\partial \lambda Ak}{\partial k} = \lambda A + \frac{\partial \lambda A}{\partial c_1} \frac{\partial c_1}{\partial k}.$$

Furthermore, taking the derivative of λA simplifies to

$$\frac{\partial \lambda A}{\partial c_1} = \lambda A \frac{1}{2c_1(1-c_1)}.$$

Then we have

$$\frac{\partial \lambda Ak}{\partial k} = \lambda A \left(1 + \frac{1}{2c_1(1-c_1)} \frac{\partial c_1}{\partial k} \right).$$

This is negative if

$$\frac{\partial c_1}{\partial k} < -2c_1(1-c_1).$$

If $\frac{\partial \lambda Ak}{\partial k} < 0$, then there exist a threshold $\bar{\sigma}_z^2$ such that an increase in k reduces information production costs more than the reduction in real investment efficiencies if $\sigma_z^2 > \bar{\sigma}_z^2$ and vice versa if $\sigma_z^2 < \bar{\sigma}_z^2$.

B Appendix - Chapter II

B.1 Proofs

B.1.1 Proof of Lemma 5

For this proof, let m of the traders, denoted by subscript j be active, and $n - m$, denoted by subscript k be inactive. This nests both the cases in the paper as $n = m$ is one alternative in this proof. α_0 , α_1 and α_2 are state-dependent, i.e. $\alpha_0(s)$, $\alpha_1(s)$, and $\alpha_2(s)$. I simplify the notation in the proof for readability.

Due to market clearing, we have the following restriction of the price:

$$\sum_j q_t^j = \sum_j \alpha_0 + \alpha_1 p_t + \alpha_2 X_t^j = m(\alpha_0 + \alpha_1 p_t) + \alpha_2 \sum_j X_t^j = 0.$$

This gives the equilibrium price

$$p_t = -\frac{\alpha_0}{\alpha_1} - \frac{\alpha_2}{m\alpha_1} \sum_j X_t^j.$$

The equilibrium trading rates are

$$q_t = 1^j (\alpha_0 + \alpha_1 p_t) + \alpha_2 I^j X_t = -\frac{\alpha_2}{m} S^j X_t + \alpha_2 I^j X_t = \alpha_2 (I^j - \frac{S^j}{m}) X_t$$

where I^j is a modified identity matrix where the diagonal values are 0 for inactive traders and 1 otherwise, 1^j is a column vector with 1 on the position for active traders and 0 otherwise, and S^j is the matrix $1^j (1^j)^T$. Trader i has the price-trading rate trade off given by

$$\begin{aligned} q_t^i &= -\sum_{j \neq i} \alpha_0 + \alpha_1 p_t + \alpha_2 X_t^j \\ &= -(m-1)\alpha_0 - (m-1)\alpha_1 p_t - \alpha_2 \sum_{j \neq i} X_t^j \end{aligned} \tag{B.1}$$

In a symmetric equilibrium in the direct revelation mechanism, the price vector P has p^u on the

positions for unrestricted traders and p^r on the position of restricted traders. This gives a price

$$p_t = p^r \sum_k E[X_t^k] + p^u \sum_j X_t^j$$

and trading rates

$$q_t = QX_t$$

in equilibrium. In a symmetric equilibrium, row i of Q for an active trader has q on the i th position, $-\frac{q}{m-1}$ on position $j \neq i$, and 0 on position k . For reports \hat{X}_t^i , trader i trades

$$q_t^i = q \hat{X}_t^i - \frac{q}{m-1} \sum_{j \neq i} X_t^j$$

and the price is

$$p_t = p^r \sum_k E[X_t^k] + p^u \sum_{j \neq i} X_t^j + p^u \hat{X}_t^i.$$

This equation rearranges to

$$\hat{X}_t^i = \frac{1}{p^u} (p_t - p^r \sum_k E[X_t^k] - p^u \sum_{j \neq i} X_t^j).$$

Trader i faces the price-trading rate trade off given by

$$\begin{aligned} q_t^i &= \frac{q}{p^u} (p_t - p^r \sum_k E[X_t^k] - p^u \sum_{j \neq i} X_t^j) - \frac{q}{m-1} \sum_{j \neq i} X_t^j \\ &= -\frac{qp^r}{p^u} \sum_k E[X_t^k] + \frac{q}{p^u} p_t - \frac{m}{m-1} q \sum_{j \neq i} X_t^j. \end{aligned} \tag{B.2}$$

The two trading mechanisms are equivalent if the following conditions hold:

$$\begin{aligned} -(m-1)\alpha_0 &= -\frac{qp^r}{p^u} \sum_k E[X_t^k] \\ -(m-1)\alpha_1 &= \frac{q}{p^u} \\ \alpha_2 &= \frac{m}{m-1} q. \end{aligned} \tag{B.3}$$

The conditions from (B.3) ensure that the equations (B.1) and (B.2) are the same. It is straightforward to show that the price and trading rates are the same when the conditions hold. The system of equations from (B.3) has a unique solution as long as both the trading mechanism and the equilibrium in the double auction are *acceptable*. As a result, there is a one-to-one map between the direct revelation mechanism and the double auction. They are therefore equivalent.

For trader i , both equilibria reveal the sum of the inventories of active traders other than trader i .

B.1.2 Proof of Lemma 6

When $q_t = Q(s_t)X_t$ and $p_t = P(s_t)X_t$, then $-\frac{b(X_t^i)^2}{2} + p_t q_t^i$ is of the form $X_t^T B^i(s_t) X_t$ for a matrix $B^i(s_t)$. Furthermore, conditional on any realization of s_t , X_t is a multivariate Ornstein-Uhlenbeck

process with state-dependent parameters. The solution of $E \left[(X_t^j)^T X_t^k | s_{[0,t]}, X_0 \right] = X_0^T C_1^{jk}(s_{[0,t]}) X_0 + C_2^{jk}(s_{[0,t]})$ where $C_1^{jk}(s_{[0,t]})$ is a matrix that depend on both the time and the realization of s_t . $C_2^{jk}(s_{[0,t]})$ is a function of time and the realization of s_t . As a result, we have

$$E \left[\int_0^\infty e^{-ru} \left(-\frac{b(X_u^i)^2}{2} + p_u q_u^i \right) ds | s_{[0,t]}, X_0 \right] = X_0 D_1^i(s_{[0,t]}) X_0 + D_2^i(s_{[0,t]})$$

as long as this is well defined. The stability assumption ensures that this is finite for all realizations of $s_{[0,t]}$ because it ensures that trading is towards equal distribution of inventories. Using the law of iterated expectations, we have

$$E \left[\int_0^\infty e^{-ru} \left(-\frac{b(X_u^i)^2}{2} + p_u q_u^i \right) ds | \mathcal{F}_{i0} \right] = E[X_0 A^i(s_0) X_0 | \mathcal{F}_{i0}] + k^i(s_0).$$

Here \mathcal{F}_{i0} contains s_0 , X_0^i , and a prior distribution over X_0^j for $j \neq i$. $A^i(s)$ is a symmetric matrix.

Next, $Y_{i0} = E[X_0 | \mathcal{F}_{i0}]$ and $\gamma_{i0} = E[(Y_{i0} - X_0)(Y_{i0} - X_0)^T | \mathcal{F}_{i0}]$ is the covariance matrix. The expected value of a quadratic form has the value

$$\begin{aligned} E[X_0^T A^i(s_0) X_0 | \mathcal{F}_{i0}] &= E[X_0^T | \mathcal{F}_{i0}] A^i(s_0) E[X_0 | \mathcal{F}_{i0}] + \text{tr}[A^i(s_0) \gamma_{i0}] \\ &= Y_{i0}^T A^i(s_0) Y_{i0} + \text{tr}[A^i(s_0) \gamma_{i0}] \end{aligned}$$

where γ_{i0} is the covariance matrix of X_0 conditional on \mathcal{F}_{i0} . This has to hold for all s_0 and gives the functional form of the value function

$$f^i(Y_{it}, s_t, \gamma_{it}) = Y_{it}^T A^i(s_t) Y_{it} + k^i(s_t) + \text{tr}[A^i(s_t) \gamma_{it}].$$

B.1.3 Proof of Lemma 7

The dynamics of inventories when trader i reports $\hat{X}_t^i = X_t^i + y^i$ and the other traders report truthfully is

$$dX_t = -Q(s_t) X_t dt - Q^i(s_t) y^i dt + \sigma(s_t) dB_t.$$

In expectation, this is

$$E[dY_{it} | \mathcal{F}_{it}] = -(Q(s_t) E[X_t | \mathcal{F}_{it}] + Q^i(s_t) y^i) dt = -(Q(s_t) Y_{it} + Q^i(s_t) y^i) dt.$$

The variance is given by

$$\begin{aligned} \gamma_{i(t+dt)} &= \text{Var}((I - Q(s_t) dt) X_t - Q^i(s_t) y^i dt + \sigma(s_t) dB_t | \mathcal{F}_{i(t+dt)}) \\ &= \text{Var}((I - Q(s_t) dt) X_t + \sigma(s_t) dB_t | \mathcal{F}_{i(t+dt)}). \end{aligned}$$

That is, y^i is in trader i 's information set at time t and therefore does not affect the variance at future periods. As a result, $d\gamma_{it}$ is independent of y^i .

The quadratic covariation only depends on new shocks. They are of the order dB_t between jumps. The effect on the trading rate from reporting different y^i is of the order dt and hence does not affect the quadratic covariation.

B.1.4 Proof of Proposition 4

Define G_t such that

$$G_t^i = \int_0^t -\frac{b(s_u)(Y_u^i)^2}{2} + (P(s_u)Y_u + p^u(s_u)y_u^i)(\underline{Q}^i(s_u)Y_u + q(s_u)y_u^i) du + e^{-rt} f^i(Y_{it}, s_t, \gamma_{it}).$$

G_t has to be a supermartingale for any arbitrary strategy y_t and a martingale for an optimal strategy. On differential form, this simplifies to

$$dG_t^i = e^{-rt} g^i(Y_{it}, s_t, \gamma_{it}, y_t^i) dt + e^{-rt} f_Y^i(Y_{it}, s_t, \gamma_{it}) \Sigma^i(s_t) dB_t + (dN_t - \lambda(s_t) dt) (E[f^i(Y_{it}', s_t', \gamma_{it}') | \mathcal{F}_{it}] - f^i(Y_{it}, s_t, \gamma_{it}))$$

where

$$g^i(Y_{it}, s_t, \gamma_{it}, y_t^i) = -\frac{b(s_t)(Y_t^i)^2}{2} + (P(s_t)Y_t + p^u(s_t)y_t^i)(\underline{Q}^i(s_t)Y_t + q(s_t)y_t^i) + \mathcal{A}(s_t, y_t^i) f^i(Y_{it}, s_t, \gamma_{it}) - r f^i(Y_{it}, s_t, \gamma_{it})$$

and N_t is a Poisson process with intensity $\lambda(s_t)$. The matrix $\Sigma^i(s_t)$ takes into account the dynamic learning process. The optimization problem ensures that $g^i(Y_{it}, s_t, \gamma_{it}, y_t^i) \leq 0$ and $g^i(Y_{it}, s_t, \gamma_{it}, 0) = 0$ as long as $p^u(s_t)q(s_t) < 0$. Hence, G_t is a martingale on $[0, T]$ if

$$E\left[\int_0^t |e^{-ru} \frac{\partial f(Y_{iu}, s_u, \gamma_{iu})}{\partial Y_{iu}} \Sigma^i(s_u)|^2 du\right] < \infty.$$

This is satisfied since $q(s) > 0$ ensures that X_t does not explode and the value function is quadratic in Y_{it} . The last part is to ensure that

$$\lim_{t \rightarrow \infty} E[e^{-rt} f(Y_{it}, s_t, \gamma_{it})] = 0.$$

This condition is satisfied for all strategies that satisfy the transversality condition since $q(s) > 0$ ensures that inventories does not blow up and the transversality condition removes potential deviations where inventories blow up. Hence, G_t is a martingale and truth telling is an equilibrium in the direct revelation mechanism.

B.1.5 Proof of Corollary 5

Both parts evaluate the effect of out-of-equilibrium behavior that trader i can choose by reporting y different from 0.

Temporary price impact

Trader i can choose trading rate by choosing y . The price and trading rate choice for trader i is given by

$$q_t^i = -\underline{Q}^i(s_t)X_t - q(s_t)y$$

and

$$p_t = P(s_t)X_t + p^u(s_t)y.$$

The temporary price impact is given by

$$\frac{\partial p_t}{\partial q_t^i} = \frac{\partial p_t}{\partial y} \frac{\partial y}{\partial q_t^i} = -\frac{p^u(s_t)}{q(s_t)}.$$

Permanent price impact

I'll do this proof for state s_1 , but it is exactly the same for state s_2 with m instead of n . Suppose that trader i reports $y_t^i \neq 0$ over the interval $[t_0, t_0 + k]$ such that $X_t = \tilde{X}_t + (X_t - \tilde{X}_t)$ where \tilde{X}_t is the equilibrium inventories. Due to symmetry and market clearing, we need $X_{t_0+k} - \tilde{X}_{t_0+k}$ to be a vector with $-x$ on position i and $\frac{x}{n-1}$ on the positions for the other $n-1$ traders. To go back to the old equilibrium trading rates after time $t_0 + k$, trader i needs to report y_t^i such that

$$Q(s_1)(X_t - \tilde{X}_t) - Q^i(s_1)y_t^i = 0.$$

This requires $y_t^i = -\frac{n}{n-1}x$ for $t > t_0 + k$. The effect on the price is $-p^u(s_1)\frac{n}{n-1}x$.

B.1.6 Proof Proposition 5

The system of equations from the first order conditions and HJB-equations consists of a system of 25 equations with 25 unknowns. The notation is as follows: $a_{ij}(s_1)$ is the element on position ij of $A^i(s_1)$. Furthermore, $a_{ij}^u(s_2)$ and $a_{ij}^r(s_2)$ are the same elements for an unrestricted and a restricted trader in state s_2 . The traders denoted by j and k can trade in state s_2 whereas trader f and g can not trade in state s_2 . The non-redundant equations from the FOCs and value function conditions, and the solution are shown at the end in section B.1.11 to fit within the width of a page. Calculations are performed with Mathematica to reduce the risk of algebraical errors. I can reduce the system to the following cubic equation:

$$b_3 q(s_2)^3 + b_2 q(s_2)^2 + b_1 q(s_2) + b_0 = 0 \quad (\text{B.4})$$

where

$$\begin{aligned} b_3 &= 4m^3(\lambda(s_1)(-n-2)nr^2(-(m+1)n+n^2+1) + \lambda(s_2)r(-(m+1)n^3 + (m(m+6)+1)n^2 - m(3m+8)n + m(m+3)+n) \\ &\quad + \lambda(s_2)^2(m-1)(m(n-3)+n) - (n-2)(n-1)r(\lambda(s_2)+r)(\lambda(s_2)(m(n-2)+n) + (n-1)nr)) \\ b_2 &= 2(m-1)m^2(\lambda(s_2)+r)(\lambda(s_1)(\lambda(s_2)r(m^2(4-3n) + m(12-(n-5)n(3n-7)) + n(-5(n-2)n-2)) \\ &\quad + (n-2)nr^2((m-5)n^2 + (7-(m-2)m)n + m-5) + 2\lambda(s_2)^2(m-1)(n-m)) \\ &\quad + (n-2)(n-1)r(\lambda(s_2)+r)(\lambda(s_2)(-3mn+8m-5n) + (m-5)(n-1)nr)) \\ b_1 &= -(m-1)^2 m(n-2)r(\lambda(s_2)+r)^2(\lambda(s_1)(2\lambda(s_2)((m+4)n^2 - (7m+3)n + m(m+4)) \\ &\quad + nr(8-3m)n^2 + (m(m+3)-14)n-3m+8) - (n-1)(\lambda(s_2)+r)((3m-8)(n-1)nr - 2\lambda(s_2)((m+4)n-5m)) \\ b_0 &= (m-1)^3(n-2)(n-1)r(\lambda(s_2)+r)^3(\lambda(s_1)+\lambda(s_2)+r)(2\lambda(s_2)(m-n) + (m-2)(n-1)nr). \end{aligned}$$

To show existence of an equilibrium, we need to show that there exist a solution to B.4 such that $p^u(s_1)q(s_1) < 0$ and $p^u(s_2)q(s_2) < 0$. Define

$$\tilde{q}(s_2) = \frac{(m-1)(\lambda(s_2)+r)(2\lambda(s_2)(m-n) + (m-2)(n-1)nr)}{2m(\lambda(s_2)(m(n-2)+n) + (n-1)nr)}.$$

$\tilde{q}(s_2) > 0$ only if $\lambda(s_2) < \bar{\lambda}$.¹⁵ By direct calculation, we get

$$\begin{aligned} & b_3 \tilde{q}(s_2)^3 + b_2 \tilde{q}(s_2)^2 + b_1 \tilde{q}(s_2) + b_0 \\ &= -\frac{\lambda(s_1)\lambda(s_2)(m-1)^4 m(m-n)^2 (\lambda(s_2)+r)^3 (\lambda(s_2)+(n-1)r) (2\lambda(s_2)+nr)^2 (2\lambda(s_2)(m-n) + (m-2)(n-1)nr)}{2(\lambda(s_2)(m(n-2)+n) + (n-1)nr)^3}. \end{aligned}$$

This is negative if $\lambda(s_2) < \bar{\lambda}$. Furthermore, $b_0 > 0$ if $\lambda(s_2) < \bar{\lambda}$. As a result, there exist a solution to equation B.4 such that $q \in (0, \tilde{q}(s_2))$ if $\lambda(s_2) < \bar{\lambda}$. $p^u(s_2) < 0$ when $q(s_2) > 0$ and the second order condition is satisfied in state s_2 . We need to ensure that it is also satisfied in state s_1 . $p^u(s_1)$ is always negative and we only need to ensure that $q(s_1) > 0$. The expression for $q(s_1)$ is

$$\begin{aligned} q(s_1) &= \frac{mq(s_2) \left\{ \lambda(s_1) \left[r \left[-(m+2)n + n^2 + 2 \right] - 2\lambda(s_2)(m-1) \right] + (n-2)(n-1)r(\lambda(s_2)+r) \right\}}{2n(\lambda(s_2)+r)(mq(s_2) + (m-1)(\lambda(s_2)+r))} \\ &+ \frac{(m-1)(n-2)(n-1)r(\lambda(s_2)+r)(\lambda(s_2)+\lambda(s_1)+r)}{2n(\lambda(s_2)+r)(mq(s_2) + (m-1)(\lambda(s_2)+r))}. \end{aligned}$$

The denominators in the expression for $q(s_1)$ are positive as long as $q(s_2) > 0$. The first fraction is multiplied by $q(s_2)$ whereas the second is always positive. When $q(s_2) \rightarrow 0$, the sum has to be positive and it is sufficient to show that this is also the case when $q(s_2) = \tilde{q}(s_2)$. Simplifying the expression gives

$$\lim_{q(s_2) \rightarrow \tilde{q}(s_2)} q(s_1) = \frac{m(n-2)(n-1)^2 r (\lambda(s_2)+r) - \lambda(s_1)(m-n) (2\lambda(s_2)(m-1) + (n-1)r(mn-2))}{2m(n-1)n(\lambda(s_2)+r)}.$$

This is positive, and all values of $q(s_2) \in (0, \tilde{q}(s_2))$ result in $q(s_1) > 0$ and both second order conditions hold if $\lambda(s_2) < \bar{\lambda}$.

B.1.7 Proof of Corollary 6

$$\lambda(s_1) = 0$$

When $\lambda(s_1) = 0$, then the parameters of the cubic equation (B.4) simplifies to

$$\begin{aligned} b_0 &= (m-1)^3 (n-2)(n-1)r(\lambda(s_2)+r)^4 (2\lambda(s_2)(m-n) + (m-2)(n-1)nr) \\ b_1 &= (m-1)^2 m(n-2)(n-1)r(\lambda(s_2)+r)^3 ((3m-8)(n-1)nr - 2\lambda(s_2)(m(n-5) + 4n)) \\ b_2 &= 2(m-1)m^2(n-2)(n-1)r(\lambda(s_2)+r)^2 (\lambda(s_2)(m(8-3n) - 5n) + (m-5)(n-1)nr) \\ b_3 &= -4m^3(n-2)(n-1)r(\lambda(s_2)+r)(\lambda(s_2)(m(n-2)+n) + (n-1)nr). \end{aligned}$$

The solutions of the cubic equation are

$$q(s_2) = \frac{(m-1)(\lambda(s_2)+r)(2\lambda(s_2)(m-n) + (m-2)(n-1)nr)}{2m(\lambda(s_2)(m(n-2)+n) + (n-1)nr)},$$

$$q(s_2) = -\frac{(m-1)(\lambda(s_2)+r)}{m},$$

and

$$q(s_2) = -\frac{(m-1)(\lambda(s_2)+r)}{2m}.$$

It is straightforward to show that only the first solution may be positive and that the condition is $\lambda(s_2) \leq \bar{\lambda}$. The other two solutions are always negative and not equilibria.

¹⁵To obtain $\tilde{q}(s_2)$, you can solve $a_{jj}(s_1) = \frac{b}{2r} \frac{N-2}{(N-1)N^2}$. This is useful for the proof of Proposition 7.

$$\lambda(s_2) = 0$$

When $\lambda(s_2) = 0$, then the parameters of the cubic equation (B.4) simplifies to

$$\begin{aligned} b_0 &= (m-2)(m-1)^3(n-2)(n-1)^2nr^5(\lambda(s_1) + r) \\ b_1 &= -(m-1)^2m(n-2)nr^4\left(\lambda(s_1)\left((8-3m)n^2 + (m(m+3)-14)n-3m+8\right) - (3m-8)(n-1)^2r\right) \\ b_2 &= 2(m-1)m^2(n-2)nr^3\left(\lambda(s_1)\left((m-5)n^2 + (7-(m-2)m)n+m-5\right) + (m-5)(n-1)^2r\right) \\ b_3 &= 4m^3(n-2)nr^2\left(\lambda(s_1)(n(m-n+1)-1) - (n-1)^2r\right). \end{aligned}$$

The solutions of the cubic equation are

$$q(s_2) = \frac{(m-2)(m-1)r}{2m},$$

$$q(s_2) = -\frac{(m-1)r}{2m},$$

and

$$q(s_2) = -\frac{(m-1)(n-1)^2r(\lambda(s_1) + r)}{\lambda(s_1)m(n(n-m-1)+1) + m(n-1)^2r}.$$

It is straightforward to show that only the first solution is positive.

B.1.8 Proof of Proposition 6

As above, the notation is as follows: $a_{jk}(s)$ is the element of $A^i(s)$ at position jk . The equilibrium conditions simplify to the following redundant equations:

FOC state s_1 :

$$\begin{aligned} 0 &= -a_{ii}(s_1) + a_{ij}(s_1) + p^u(s_1) \\ 0 &= (n-2)p^u(s_1) - 2(n-1)(a_{ij}(s_1) - a_{jk}(s_1)) \end{aligned}$$

FOC state s_2 :

$$\begin{aligned} 0 &= -a_{ii}(s_2) + a_{ij}(s_2) + p^u(s_2) \\ 0 &= (n-2)p^u(s_2) - 2(n-1)(a_{ij}(s_2) - a_{jk}(s_2)) \end{aligned}$$

Value function state s_1 :

$$\begin{aligned} 0 &= a_{ii}(s_1)(\lambda(s_1) + 2q(s_1) + r) - 2a_{ij}(s_1)q(s_1) + \frac{b(s_1)}{2} - a_{ii}(s_2)\lambda(s_1) - p^u(s_1)q(s_1) \\ 0 &= \frac{q(s_1)(a_{ij}(s_1) - a_{ii}(s_1))}{n-1} + q(s_1)(a_{ij}(s_1) - a_{jk}(s_1)) + a_{ij}(s_1)(\lambda(s_1) + r) - a_{ij}(s_2)\lambda(s_1) - \frac{(n-2)p^u(s_1)q(s_1)}{2(n-1)} \\ 0 &= \frac{2q(s_1)(a_{jk}(s_1) - a_{ij}(s_1))}{n-1} + a_{jk}(s_1)(\lambda(s_1) + r) - a_{jk}(s_2)\lambda(s_1) + \frac{p^u(s_1)q(s_1)}{n-1} \end{aligned}$$

Value function state s_2 :

$$\begin{aligned}
0 &= -a_{ii}(s_1)\lambda(s_2) + \frac{b(s_2)}{2} + a_{ii}(s_2)(\lambda(s_2) + 2q(s_2) + r) - 2a_{ij}(s_2)q(s_2) - p^u(s_2)q(s_2) \\
0 &= -a_{ij}(s_1)\lambda(s_2) + \frac{q(s_2)(a_{ij}(s_2) - a_{ii}(s_2))}{n-1} + q(s_2)(a_{ij}(s_2) - a_{jk}(s_2)) + a_{ij}(s_2)(\lambda(s_2) + r) - \frac{(n-2)p^u(s_2)q(s_2)}{2(n-1)} \\
0 &= -a_{jk}(s_1)\lambda(s_2) + \frac{2q(s_2)(a_{jk}(s_2) - a_{ij}(s_2))}{n-1} + a_{jk}(s_2)(\lambda(s_2) + r) + \frac{p^u(s_2)q(s_2)}{n-1}
\end{aligned}$$

The unique solution is

$$\begin{aligned}
q(s_1) &= \frac{b(s_1)(n-2)(n-1)r(\lambda(s_1) + \lambda(s_2) + r)}{2n(b(s_1)(\lambda(s_2) + r) + b(s_2)\lambda(s_1))} \\
q(s_2) &= \frac{b(s_2)(n-2)(n-1)r(\lambda(s_1) + \lambda(s_2) + r)}{2n(b(s_1)\lambda(s_2) + b(s_2)(\lambda(s_1) + r))} \\
p^u(s_1) &= -\frac{b(s_1)(\lambda(s_2) + r) + b(s_2)\lambda(s_1)}{nr(\lambda(s_1) + \lambda(s_2) + r)} \\
p^u(s_2) &= -\frac{b(s_1)\lambda(s_2) + b(s_2)(\lambda(s_1) + r)}{nr(\lambda(s_1) + \lambda(s_2) + r)} \\
a_{ii}(s_1) &= -\frac{(3n-2)(b(s_1)(\lambda(s_2) + r) + b(s_2)\lambda(s_1))}{2n^2r(\lambda(s_1) + \lambda(s_2) + r)} \\
a_{ij}(s_1) &= -\frac{(n-2)(b(s_1)(\lambda(s_2) + r) + b(s_2)\lambda(s_1))}{2n^2r(\lambda(s_1) + \lambda(s_2) + r)} \\
a_{jk}(s_1) &= \frac{(n-2)(b(s_1)(\lambda(s_2) + r) + b(s_2)\lambda(s_1))}{2(n-1)n^2r(\lambda(s_1) + \lambda(s_2) + r)} \\
a_{ii}(s_2) &= -\frac{(3n-2)(b(s_1)\lambda(s_2) + b(s_2)(\lambda(s_1) + r))}{2n^2r(\lambda(s_1) + \lambda(s_2) + r)} \\
a_{ij}(s_2) &= -\frac{(n-2)(b(s_1)\lambda(s_2) + b(s_2)(\lambda(s_1) + r))}{2n^2r(\lambda(s_1) + \lambda(s_2) + r)} \\
a_{jj}(s_2) &= \frac{(n-2)(b(s_1)\lambda(s_2) + b(s_2)(\lambda(s_1) + r))}{2(n-1)n^2r(\lambda(s_1) + \lambda(s_2) + r)}.
\end{aligned}$$

This simplifies to the matrix form from the Proposition.

B.1.9 Proof of Corollary 7

The ratio of the temporary price impact of state s_1 relative to state s_2 is given by

$$\frac{I(s_1)}{I(s_2)} = \frac{b(s_2)}{b(s_1)} \left(\frac{b(s_1)(\lambda(s_2) + r) + b(s_2)\lambda(s_1)}{b(s_1)(\lambda(s_2)) + b(s_2)(\lambda(s_1) + r)} \right)^2.$$

If r is sufficiently small, then this approaches $\frac{b(s_2)}{b(s_1)}$ and if it is sufficiently large, then it approaches $\frac{b(s_1)}{b(s_2)}$.

There is a unique solution such that $\frac{I(s_2)}{I(s_1)} = 1$ when

$$r = \sqrt{\frac{b(s_1)}{b(s_2)}} \left(1 + \frac{b(s_2)}{b(s_1)} \frac{\lambda(s_1)}{\lambda(s_2)} \right) \lambda(s_2).$$

If r is lower than this threshold, then the temporary price impact is larger in state s_1 such that price impact and volume are negatively correlated. The opposite is the case when r is above the threshold. The size of the region where the price impact and volume are positively correlated always increases in $\frac{b(s_2)}{b(s_1)}$ when it is sufficiently large. If $\lambda(s_1) \geq \lambda(s_2)$, then the size of the region is always increasing in $\frac{b(s_2)}{b(s_1)}$.

B.1.10 Proof of Proposition 7

The assumption on no initial inventories and perfect knowledge about X_0 ensures that the proof boils down to calculating $k^i(s)$. From equation 2.4 this simplifies to the three equations

$$(r + \lambda(s_1))k^i(s_1) = \sigma^2(s_1)\text{tr}[A^i(s_1)] + \lambda(s_1)(\pi k^{i,u}(s_2) + (1 - \pi)k^{i,r}(s_1)),$$

$$(r + \lambda(s_2))k^{i,u}(s_1) = \sigma^2(s_2)\text{tr}[A^{i,u}(s_2)] + \lambda(s_2)k^i(s_1)$$

and

$$(r + \lambda(s_2))k^{i,r}(s_1) = \sigma^2(s_2)\text{tr}[A^{i,r}(s_2)] + \lambda(s_2)k^i(s_1).$$

Here, the superscript r denotes that trader i is restricted in state s_2 and u for an unrestricted trader. The probability of being restricted in state s_2 is $\pi \in (0, 1)$. This is a linear system with a solution of the form

$$k^i(s) = c_1\sigma^2(s_1)^2\text{tr}[A^i(s_1)] + c_2\sigma^2(s_2)\text{tr}[A^{i,u}(s_2)] + c_3\sigma^2(s_2)\text{tr}[A^{i,r}(s_2)]$$

where the three constants c_1 , c_2 , and c_3 are positive.

There exist an equilibrium where the value of $\text{tr}[A^i(s_1)]$ is larger in the constrained equilibrium than in an equilibrium where all traders trade all the time¹⁶. To see this, $a_{ii}(s_1) = \frac{b(2-3n)}{2n^2r}$ in both a constrained and in an unconstrained equilibrium. Hence, we only have to compare a_{jj} in the unconstrained equilibrium of Result 1 and $a_{jj}(s_1)$ in the constrained equilibrium of Proposition 5. In the existence part of the proof of that proposition, I show that there exist a solution where $q(s_2) < \tilde{q}(s_2)$ where $\tilde{q}(s_2)$ is the unique trading rate in state s_2 such that a_{jj} in the unconstrained equilibrium of Result 1 and $a_{jj}(s_1)$ in the constrained equilibrium of Proposition 5 are equal. The only remaining part is to show that $a_{jj}(s_1)$ is decreasing in $q(s_2)$. We have the derivative with respect to $q(s_2)$ given by

$$\frac{\partial a_{jj}(s_1)}{\partial q(s_2)} = -\frac{b(m-1)m^2(n-2)(\lambda(s_2) + r)(2\lambda(s_2) + nr)}{4\lambda(s_2)nr(n-m)(\lambda(s_2)(m-1) + m(q(s_2) + r) - r)^2} < 0.$$

This derivative is negative for all parameters. As a result, there always exist a constrained equilibrium where $\text{tr}[A^i(s_1)]$ is larger in the constrained equilibrium than in the unconstrained equilibrium. Hence, if $\sigma^2(s_1)$ is sufficiently large relative to $\sigma^2(s_2)$, the change of $\text{tr}[A^i(s_1)]$ dominates the other terms when we compare welfare in the unconstrained and constrained equilibria.

¹⁶The values for elements of $A^i(s_1)$ follows after this proof as the part with the algebra from Proposition 5. See section B.1.11. The solutions are at the end of that part.

B.1.11 Algebra Proposition 5

First order condition state s_1 :

$$0 = 2q(s_1)(-a_{ii}(s_1) + a_{ij}(s_1) + p^u(s_1))$$

$$0 = \frac{q(s_1)(-2a_{ij}(s_1)(n-1) + 2a_{jj}(s_1) + (n-2)(2a_{jk}(s_1) + p^u(s_1)))}{n-1}$$

First order condition in state s_2 :

$$0 = 2q(s_2)(-a_{ii}^u(s_2) + a_{ij}^u(s_2) + p^u(s_2))$$

$$0 = \frac{q(s_2)(-2a_{ij}^u(s_2)(m-1) + 2a_{jj}^u(s_2) + (m-2)(2a_{jk}^u(s_2) + p^u(s_2)))}{m-1}$$

$$0 = q(s_2)(-2a_{if}^u(s_2) + 2a_{jf}^u(s_2) + p^r(s_2))$$

The value function conditions in state s_1 :

$$0 = \frac{\lambda(s_1)(a_{ii}(s_1)n - a_{ii}^u(s_2)m + a_{ii}^r(s_2)m - a_{ii}^r(s_2)n)}{n} + a_{ii}(s_1)(2q(s_1) + r) - q(s_1)(2a_{ij}(s_1) + p^u(s_1)) + \frac{b}{2}$$

$$0 = a_{ij}(s_1) \left(\lambda(s_1) + \frac{nq(s_1)}{n-1} + r \right)$$

$$- \frac{nq(s_1)(2a_{ii}(s_1) + 2a_{jj}(s_1) + (n-2)(2a_{jk}(s_1) + p^u(s_1))) + 2\lambda(s_1)(a_{ij}^u(s_2)(m-1)m + (m-n)(a_{if}^r(s_2)(m-n+1) - m(a_{if}^u(s_2) + a_{ij}^r(s_2))))}{2(n-1)n}$$

$$0 = \frac{nq(s_1)(-2a_{ij}(s_1) - 2a_{jk}(s_1)n + 4a_{jk}(s_1) + p^u(s_1)) - \lambda(s_1)(a_{jj}^u(s_2)(m-1)m + (m-n)(a_{ff}^r(s_2)(m-n+1) - m(a_{ff}^u(s_2) + a_{jj}^r(s_2))))}{(n-1)n}$$

$$+ a_{jj}(s_1)(\lambda(s_1) + 2q(s_1) + r)$$

$$0 = - \frac{2q(s_1)(a_{ij}(s_1) + a_{jj}(s_1) - 2a_{jk}(s_1))}{n-1} + a_{jk}(s_1)(\lambda(s_1) + r) + \frac{p^u(s_1)q(s_1)}{n-1}$$

$$+ \frac{\lambda(s_1)((m-n)(a_{fg}^r(s_2)(m-n+1)(m-n+2) - m(-2a_{jf}^u(s_2)(m-1) + m(a_{fg}^u(s_2) + 2a_{jf}^r(s_2) - a_{jk}^r(s_2)) - n(a_{fg}^u(s_2) + 2a_{jf}^r(s_2)) + a_{fg}^u(s_2) + 2a_{jf}^r(s_2) + a_{jk}^r(s_2))) - a_{jk}^u(s_2)(m-2)(m-1)m)}{(n-2)(n-1)n}$$

The value function for unrestricted traders conditions in state s_2 :

$$\begin{aligned}
0 &= \lambda(s_2)(a_{ii}^u(s_2) - a_{ii}(s_1)) + \frac{b}{2} + q(s_2)(2a_{ii}^u(s_2) - 2a_{ij}^u(s_2) - p^u(s_2)) + a_{ii}^u(s_2)r \\
0 &= \frac{(m-1)\left(\lambda(s_2)(a_{jj}^u(s_2) - a_{jj}(s_1)) + a_{jj}^u(s_2)r\right) + q(s_2)(p^u(s_2) - 2(a_{ij}^u(s_2) - a_{ij}^u(s_2))m + a_{ij}^u(s_2) + a_{jk}^u(s_2)(m-2))}{m-1} \\
0 &= \lambda(s_2)(a_{ff}^u(s_2) - a_{jj}(s_1)) + a_{ff}^u(s_2)r \\
0 &= \frac{2(m-1)\left(\lambda(s_2)(a_{ij}^u(s_2) - a_{ij}(s_1)) + a_{ij}^u(s_2)r\right) - q(s_2)(2(a_{ii}^u(s_2) - a_{ij}^u(s_2))m + a_{ij}^u(s_2) + a_{jk}^u(s_2)(m-2)) + (m-2)p^u(s_2)}{2(m-1)} \\
0 &= \lambda(s_2)(a_{if}^u(s_2) - a_{ij}(s_1)) + q(s_2)\left(a_{if}^u(s_2) - a_{jf}^u(s_2) - \frac{p^r(s_2)}{2}\right) + a_{if}^u(s_2)r \\
0 &= \frac{2(m-1)\left(\lambda(s_2)(a_{jf}^u(s_2) - a_{jk}(s_1)) + a_{jf}^u(s_2)r\right) + q(s_2)(-2a_{if}^u(s_2) + 2a_{jf}^u(s_2) + p^r(s_2))}{2(m-1)} \\
0 &= \frac{(m-1)\left(\lambda(s_2)(a_{jk}^u(s_2) - a_{jk}(s_1)) + a_{jk}^u(s_2)r\right) + q(s_2)(p^u(s_2) - 2(a_{ij}^u(s_2) + a_{jj}^u(s_2) - 2a_{jk}^u(s_2)))}{m-1} \\
0 &= \lambda(s_2)(a_{fg}^u(s_2) - a_{jk}(s_1)) + a_{fg}^u(s_2)r
\end{aligned}$$

The value function for restricted traders conditions in state s_2 :

$$\begin{aligned}
0 &= \lambda(s_2)(a_{ii}^r(s_2) - a_{ii}(s_1)) + \frac{b}{2} + a_{ii}^r(s_2)r \\
0 &= \lambda(s_2)(a_{jj}^r(s_2) - a_{jj}(s_1)) + 2q(s_2)(a_{jj}^r(s_2) - a_{ij}^r(s_2)) + a_{jj}^r(s_2)r \\
0 &= \lambda(s_2)(a_{ff}^r(s_2) - a_{jj}(s_1)) + a_{ff}^r(s_2)r \\
0 &= \lambda(s_2)(a_{if}^r(s_2) - a_{ij}(s_1)) + a_{if}^r(s_2)r \\
0 &= \lambda(s_2)(a_{ij}^r(s_2) - a_{ij}(s_1)) + a_{ij}^r(s_2)r \\
0 &= \lambda(s_2)(a_{jf}^r(s_2) - a_{jk}(s_1)) + a_{jf}^r(s_2)r \\
0 &= \lambda(s_2)(a_{ij}^r(s_2) - a_{jk}(s_1)) + \frac{2q(s_2)(a_{ij}^r(s_2) - a_{jj}^r(s_2))}{m-1} + a_{ij}^r(s_2)r \\
0 &= \lambda(s_2)(a_{ff}^r(s_2) - a_{jk}(s_1)) + a_{ff}^r(s_2)r
\end{aligned}$$

I solve the system of equations with Mathematica in the following way:

- (1) First I solve all equations except for the last two conditions from the HJB-equation in state s_1 for all the unknowns except for $p^u(s_1)$ and $q(s_2)$. The choice of which two equations to solve is arbitrary, but this choice works well, and the equations are solved quickly. I impose that there is trade in both states and divide the first-order conditions by the trading rates $q(s_1)$ or $q(s_2)$. There are always equilibria with no-trade or trade in only one of the states.

(2) Next, I solve the remaining two equations for $p^u(s_1)$ and $q(s_2)$. All solutions have $p^u(s_1) = -\frac{b}{nr}$.

(3) In the last step, I simplify the two remaining equations with this solution for $p^u(s_1)$, and they both simplify to a cubic equation times something different from 0.

The solution as a function of $q(s_2)$ is given by:

$$q(s_1) = \frac{mq(s_2)(\lambda(s_1)(r - (m+2)n + n^2 + 2) - 2\lambda(s_2)(m-1)) + (n-2)(n-1)r(\lambda(s_2)+r) + (m-1)(n-2)(n-1)r(\lambda(s_2)+r)(\lambda(s_2)+\lambda(s_1)+r)}{2n(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda(s_2)+r))}$$

$$p^u(s_1) = -\frac{b}{nr}$$

$$p^u(s_2) = -\frac{b(m-1)(2\lambda(s_2)+nr)}{2nr(mq(s_2)+(m-1)(\lambda(s_2)+r))}$$

$$p^r(s_2) = \frac{b(2mnq(s_2) - (m-1)(2\lambda(s_2)(m-n) + (m-2)nr))}{2nr(m-n)(mq(s_2)+(m-1)(\lambda(s_2)+r))}$$

$$a_{ii}(s_1) = \frac{b(2-3n)}{2n^2r}$$

$$a_{ij}(s_1) = -\frac{b(n-2)}{2n^2r}$$

$$a_{jj}(s_1) = \frac{b(n-2)(m-1)(\lambda(s_2)+r)((m-2)n^2r + 2\lambda(s_2)(m-n) - 2mq(s_2)(\lambda(s_2)(m(n-1)+n) + n^2r))}{4\lambda(s_2)n^2r(n-m)(mq(s_2)+(m-1)(\lambda_1+r))}$$

$$a_{jk}(s_1) = \frac{b(m-1)(\lambda(s_2)+r)((m-2)n^2r + 4\lambda(s_2)(m-n) - 2mq(s_2)(\lambda(s_2)((m+2)n-2m) + n^2r))}{4\lambda(s_2)n^2r(m-n)(mq(s_2)+(m-1)(\lambda_1+r))}$$

$$a_{ii}^u(s_2) = -\frac{b(q(s_2)(\lambda(s_2)((m+2)n-2m) + n^2r) + (m-1)(\lambda(s_2)+r)(n^2r + \lambda(s_2)(3n-2)))}{2n^2r(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda_1+r))}$$

$$a_{ij}^u(s_2) = -\frac{b(q(s_2)(\lambda(s_2)((m+2)n-2m) + n^2r) + \lambda(s_2)(m-1)(n-2)(\lambda(s_2)+r))}{2n^2r(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda(s_2)+r))}$$

$$a_{if}^u(s_2) = -\frac{b\lambda(s_2)(n-2)}{2n^2r(\lambda(s_2)+r)}$$

$$a_{jj}^u(s_2) = -\frac{b(-2(m-1)q(s_2)(\lambda(s_2)+r)(\lambda(s_2)(m^2((n-3)n+6) + m(n-8)n+2n^2) + n^2r(m-3)m+n) + 4mq(s_2)^2(m-n)(\lambda(s_2)((m+2)n-2m) + n^2r) + (m-1)^2(n-2)(\lambda(s_2)+r)^2((m-2)n^2r + 2\lambda(s_2)(m-n)))}{4n^2r(m-n)(\lambda(s_2)+r)(2m^2q(s_2)^2 + 3(m-1)mq(s_2)(\lambda(s_2)+r) + (m-1)^2(\lambda_1+r)^2)}$$

$$a_{ff}^u(s_2) = \frac{b(n-2)(m-1)(\lambda(s_2)+r)((m-2)n^2r + 2\lambda(s_2)(m-n) - 2mq(s_2)(\lambda(s_2)(m(n-1)+n) + n^2r))}{4n^2r(n-m)(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda_1+r))}$$

$$a_{jk}^u(s_2) = \frac{b(2(m-1)q(s_2)(\lambda(s_2)+r)(\lambda(s_2)(6m^2 - (m+8)mn + 2n^2) + n^2r(m(m-n-2)+n)) - 4mq(s_2)^2(m-n)(\lambda(s_2)((m+2)n-2m) + n^2r) + (m-1)^2(\lambda_1+r)^2((m-2)n^2r + 4\lambda(s_2)(m-n)))}{4n^2r(m-n)(\lambda(s_2)+r)(2m^2q(s_2)^2 + 3(m-1)mq(s_2)(\lambda(s_2)+r) + (m-1)^2(\lambda(s_2)+r)^2)}$$

$$a_{fg}^u(s_2) = \frac{b(m-1)(\lambda(s_2)+r)((m-2)n^2r + 4\lambda(s_2)(m-n) - 2mq(s_2)(\lambda(s_2)((m+2)n-2m) + n^2r))}{4n^2r(m-n)(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda_1+r))}$$

$$a_{jf}^u(s_2) = \frac{b(m-1)(\lambda(s_2)+r)((m-2)n^2r + 4\lambda(s_2)(m-n) - 2mq(s_2)(\lambda(s_2)((m+2)n-2m) + n^2r))}{4n^2r(m-n)(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda_1+r))}$$

$$a_{ii}^r(s_2) = -\frac{b(n^2r + \lambda(s_2)(3n-2))}{2n^2r(\lambda(s_2)+r)}$$

$$a_{ij}^r(s_2) = -\frac{b\lambda(s_2)(n-2)}{2n^2r(\lambda(s_2)+r)}$$

$$a_{if}^r(s_2) = -\frac{b\lambda(s_2)(n-2)}{2n^2r(\lambda(s_2)+r)}$$

$$a_{jj}^r(s_2) = -\frac{b(4mq(s_2)^2(\lambda(s_2)(-2m^2 - (m+1)n^2 + (m+4)mn) + n^2r(m-n+1)) - 2(m-1)q(s_2)(\lambda(s_2)+r)(\lambda(s_2)(m^2((n-3)n+6) + m(n-8)n+2n^2) + n^2r(m^2-3m+2n-2)) + (m-1)^2(n-2)(\lambda(s_2)+r)^2((m-2)n^2r + 2\lambda(s_2)(m-n)))}{4n^2r(m-n)(\lambda(s_2)+r)(2m^2q(s_2)^2 + 3(m-1)mq(s_2)(\lambda(s_2)+r) + (m-1)^2(\lambda(s_2)+r)^2)}$$

$$a_{ff}^r(s_2) = \frac{b(n-2)(m-1)(\lambda(s_2)+r)((m-2)n^2r + 2\lambda(s_2)(m-n) - 2mq(s_2)(\lambda(s_2)(m(n-1)+n) + n^2r))}{4n^2r(n-m)(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda_1+r))}$$

$$\begin{aligned}
a_{jk}^r(s_2) &= \frac{b\left(4mq(s_2)^2\left(\lambda(s_2)\left(2m^2+(m+1)n^2-(m+4)mn\right)+n^2r(-m+n-1)\right)-2(m-1)q(s_2)(\lambda(s_2)+r)\left(\lambda(s_2)\left(-6m^2+(m+8)mn-2n^2\right)+n^2r(m(-m+n+2)-2n+2)\right)+(m-1)^2(\lambda(s_2)+r)^2\left((m-2)n^2r+4\lambda(s_2)(m-n)\right)\right)}{4n^2r(m-n)(\lambda(s_2)+r)\left(2m^2q(s_2)^2+3(m-1)mq(s_2)(\lambda(s_2)+r)+(m-1)^2(\lambda_1+r)^2\right)} \\
a_{fg}^r(s_2) &= \frac{b\left((m-1)(\lambda(s_2)+r)\left((m-2)n^2r+4\lambda(s_2)(m-n)\right)-2mq(s_2)\left(\lambda(s_2)\left((m+2)n-2m\right)+n^2r\right)\right)}{4n^2r(m-n)(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda_1+r))} \\
a_{jf}^r(s_2) &= \frac{b\left((m-1)(\lambda(s_2)+r)\left((m-2)n^2r+4\lambda(s_2)(m-n)\right)-2mq(s_2)\left(\lambda(s_2)\left((m+2)n-2m\right)+n^2r\right)\right)}{4n^2r(m-n)(\lambda(s_2)+r)(mq(s_2)+(m-1)(\lambda_1+r))}.
\end{aligned}$$

C Appendix - Chapter III

C.1 Extensions

C.1.1 Welfare simulations

To evaluate the effect of anonymity on total welfare, I simulate the values of b_i uniformly between b_m and b_n and let σ_i^2 be the same for all traders. For every simulation, I calculate both the price impact in the fully anonymous market (λ_a) and the price impact such that welfare is the same in the anonymous market as in the fully transparent market ($\hat{\lambda}_a$). Total welfare in the anonymous market is higher if λ_a is lower than $\hat{\lambda}_a$ since the incentives for bid shading are lower, and total inventory costs will be lower. Furthermore, to simplify comparison between different realizations of the simulated b_i , I calculate $\frac{\lambda_a}{b} = \frac{N}{N-1} - h_a$ and $\frac{\hat{\lambda}_a}{b} = \frac{N}{N-1} - \hat{h}_a$. If traders are homogeneous, then $h_a = \hat{h}_a = 0$ and the traders are indifferent. Otherwise, this is no longer the case.

As a benchmark, I compare the results to the case when $b_1 = b_m$ and $b_i = b_n$ for all $i > 1$. It is straightforward to show that the anonymous market has higher welfare in this example by calculating $\frac{\hat{\lambda}_a}{\lambda_a} \leq 1$ which holds with equality if and only if $b_m = b_n$ and strict inequality otherwise.

All my numerical results have $h_a > 0$, $\hat{h}_a > 0$, and $\frac{\hat{h}_a}{h_a} < 1$. Furthermore, for all simulations, the ratio between \hat{h}_a and h_a is highest when the parameters are of the form $b_1 = b_m$ and $b_i = b_n$ for all $i > 1$.

One realization of the simulations is as follows: I simulate the model 1 000 000 times with $b_i \in [1, 2]$, $\sigma_i^2 = 1$ and 4 traders. Out of those simulations, the maximum value of $\frac{\hat{h}_a}{h_a}$ is 0.9977 with the following inventory holding cost parameters: $b_1 = 1.004$, $b_2 = 1.978$, $b_3 = 1.947$, and $b_4 = 1.996$. This does indeed suggest that the structure that makes $\frac{\hat{h}_a}{h_a}$ largest is of the form discussed above, and it is possible to show that welfare is higher in the anonymous market in this case, but the algebra is tedious.

C.1.2 Endogenously noisy order flow

The noise of Proposition 8 can be made endogenous if it is costly for traders to reveal their identity and they have to pay this cost before observing the realization of their trading needs. Suppose that trader i needs to pay a cost $c_i > 0$ to be able to reveal his or her identity. Two interpretations of this cost is a fixed cost to keep a relationship with a dealer in an OTC market or to be present at a trading

venue where you can trade without anonymity. Define $\tilde{\lambda}_i$ as

$$\tilde{\lambda}_i = \frac{b_i(b_i + \bar{b}(\bar{b} + b_i)\frac{\sigma_i^2}{c_i})}{(b_i^2 - \bar{b}^2)\frac{\sigma_i^2}{c_i} - 2b_i}.$$

Lemma 9. *Traders that obtain a price impact that is larger than $\tilde{\lambda}_i$ by trading anonymously prefer to reveal their identity.*

Proof: See Appendix C.2.4.

Traders are better off by revealing their identity if the anonymous price impact is larger than $\tilde{\lambda}_i$. In addition to differences in how fast a trader would like to trade, this setting also considers how large positions they usually want to offload and the cost of being able to reveal their identity.

Proposition 10. *Every pure strategy equilibrium is a threshold equilibrium where traders with $\tilde{\lambda}_i < \lambda$ reveal their identities.*

Proof: See Appendix C.2.5.

An empirical prediction is that traders with larger b_i reveal their identity, but only if they have sufficiently large trading needs through σ_i^2 or sufficiently low costs of revealing their identity. One such example would be a large, passive investor who follows an index and has large inflows or outflows over time. One issue with Proposition 10 is that there may be multiple equilibria. It is also straightforward to find an example where any threshold is a Linear Bayesian Nash Equilibrium. To see this, fix holding costs b_i and set $\frac{\sigma_i^2}{c_i}$ for each trader such that trader i is indifferent between being anonymous or not if all traders $j < i$ reveal their identity. With this setting, all thresholds will be equilibria due to strategic complementarities in the decision to reveal identities. One alternative that obtains robust predictions is global games. Let $\sigma_i^2 = \bar{\sigma}_i^2 V$ where $V \in [0, \infty)$ and $\bar{\sigma}_i^2$ is constant for all i . Every trader observes a noisy signal of V on the form $s_i = V + k\epsilon_i$ where $\epsilon_i \sim U(-1, 1)$ and $k \rightarrow 0$.

Proposition 11. *There is a unique equilibrium where traders reveal their identity if $s_i > \bar{s}_i$ for a threshold \bar{s}_i with one exception. Traders with the lowest holding cost never reveal their identity.*

Proof: See Appendix C.2.6.

Proposition 11 offers more empirical predictions. We would expect more traders to reveal their identity by trading at non-anonymous venues at periods with large inventory shocks. Moreover, we would expect that the price impacts on both types of venues increase during these periods. The reason is that the average trader on both types of trading venues becomes more patient in these periods. Suppose that inventory shocks are larger when the market is more volatile. Then we would expect that volume moves to trading-venues with less anonymity in these periods and that the price impact increases on all venues.

C.2 Proofs

C.2.1 Proof of Lemma 8

Part (i)

The marginal inventory cost for a trader with inventory X_{i2} at time 2 is $-b_i X_{i2}$. The marginal cost of buying additional units of the asset is P_2 . Since the price is the same for all traders, this means that time 2 position for trader i in terms of the position of trader 1 is $X_{2i} = \frac{b_1}{b_i} X_{21}$. The total demand is $\sum_i X_{2i} = \sum_i \frac{b_1}{b_i} X_{21} = b_1 X_{21} \frac{1}{\bar{b}}$. Use $-b_1 X_{21} = P_2$ and $\sum_i X_{2i} = \bar{X}$ and we get $\bar{X} = -P_2 \frac{1}{\bar{b}}$. Rearrange and get $P_2 = -\bar{b}\bar{X}$.

Part (ii)

First note that X_{i2} is independent of Y_{i1} . We can rewrite the expected utility of investor i as

$$\begin{aligned} E[U_i|X_i, Y_{i1}] &= E\left[-\frac{b_i(X_i + Y_{i1})^2}{2} - P_1 Y_{i1} - \frac{b_i(X_{i2})^2}{2} - P_2(X_{i2} - X_i - Y_{i1})|X_i, Y_{i1}\right] \\ &= -\frac{b_i(X_i + Y_{i1})^2}{2} - (E[P_1|X_i, Y_{i1}] - E[P_2|X_i, Y_{i1}])Y_{i1} + A_i \\ &= -\frac{b_i(X_i + Y_{i1})^2}{2} - (\lambda_i Y_{i1} + \bar{b}X_i)Y_{i1} + A_i \end{aligned} \quad (\text{C.1})$$

where $A_i = E\left[-\frac{b_i(X_{i2})^2}{2} - P_2(X_{i2} - X_i)|X_i, Y_{i1}\right]$, $E[P_1|X_i, Y_{i1}] = \lambda_i Y_{i1}$ by the assumption of linear price functions and $E[\bar{X}] = 0$ and $E[P_2|X_i, Y_{i1}] = -\bar{b}X_i$. The first order condition is given by

$$-b_i(X_i + Y_{i1}) - 2\lambda_i Y_{i1} - \bar{b}X_i = 0$$

and gives $Y_{i1} = -\frac{b_i + \bar{b}}{b_i + 2\lambda_i} X_i$. The second order condition is $b_i + 2\lambda_i > 0$. If we plug the first order condition into (C.1) we get

$$E[U_i|X_i] = \frac{(\bar{b}^2 + 2\bar{b}b_i - 2b_i\lambda_i)X_i^2}{2(b_i + 2\lambda_i)} + A_i$$

and

$$\frac{\partial E[U_i|X_i]}{\partial \lambda_i} = -\frac{(\bar{b} + b_i)^2 X_i^2}{(b_i + 2\lambda_i)^2} < 0.$$

Part (iii)

A competitive market maker sets the price $P_1 = E[P_2|\mathcal{F}_M] = -\bar{b}E[\bar{X}|\mathcal{F}_M]$ where \mathcal{F}_M is the information set for the market maker at time 1. Define the set \mathcal{M} such that every trader $i \in \mathcal{M}$ reveals his order flow and every trader $i \notin \mathcal{M}$ stays anonymous. Define $\bar{Y} = \sum_{j \notin \mathcal{M}} Y_{j1}$. The market maker has to calculate

$$E[\bar{X}|Y_{i1} \forall i \in \mathcal{M}, \bar{Y}] = \sum_{i \in \mathcal{M}} E[X_i|Y_{i1}] + \sum_{j \notin \mathcal{M}} E[X_j|\bar{Y}].$$

The order flow of trader i at time 1 is on the form $Y_{i1} = -a_i X_i$ where a_i depends on λ_i or λ depending on the decision to reveal or not reveal order flow information. All the signals are normal and it simplifies to

$$\begin{aligned} E[\bar{X}|Y_{i1} \forall i \in \mathcal{M}, \bar{Y}] &= \sum_{i \in \mathcal{M}} \frac{\text{cov}(-Y_{i1}, X_i)}{\text{var}(Y_{i1})} + \sum_{j \notin \mathcal{M}} \frac{\text{cov}(\bar{Y}, X_j)}{\text{var}(\bar{Y})} \\ &= \sum_{i \in \mathcal{M}} -\frac{1}{a_i} Y_{i1} - \frac{\sum_{j \notin \mathcal{M}} \sigma_j^2 a_j}{\sum_{j \notin \mathcal{M}} \sigma_j^2 a_j^2} \bar{Y}. \end{aligned}$$

The price impact of a trader who reveals his order flow solves the fixed point equation

$$\lambda_i = \bar{b} \frac{1}{a_i}$$

and has the solution $\lambda_i = \frac{\bar{b} b_i}{b_i - \bar{b}}$ with $\frac{\partial \lambda_i}{\partial b_i} < 0$. If trader i stays anonymous, his price impact solves the fixed point equation

$$\lambda = \bar{b} \frac{\sum_{j \notin \mathcal{M}} \sigma_j^2 a_j}{\sum_{j \notin \mathcal{M}} \sigma_j^2 a_j^2}. \quad (\text{C.2})$$

Let λ be the solution to $\lambda = \bar{b} \frac{1}{a_0}$ for a trader with inventory cost b_0 and define k_i to satisfy $k_i a_0 = a_i$. Plugging this into (C.2) gives

$$\lambda = \bar{b} \frac{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j a_0}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2 a_0^2} = \bar{b} \frac{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j}{a_0 \sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2} = \bar{b} \frac{1}{a_0} K.$$

where $K = \frac{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2}$. To be an equilibrium, we need $K = 1$. Next we can show that $\frac{\partial K}{\partial b_0} > 0$ in any equilibrium.

$$\begin{aligned} \frac{\partial K}{\partial b_0} &= \frac{\sum_{j \notin \mathcal{M}} \sigma_j^2 \frac{\partial k_j}{\partial b_0}}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2} - \frac{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2} \frac{\sum_{j \notin \mathcal{M}} 2\sigma_j^2 \frac{\partial k_j}{\partial b_0} k_j}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2} \\ &= \frac{1}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2} \left(\sum_{j \notin \mathcal{M}} \sigma_j^2 \frac{\partial k_j}{\partial b_0} - K \sum_{j \notin \mathcal{M}} 2\sigma_j^2 \frac{\partial k_j}{\partial b_0} k_j \right) \\ &= \frac{1}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2} \left(\sum_{j \notin \mathcal{M}} \sigma_j^2 \frac{\partial k_j}{\partial b_0} (1 - 2k_j) \right). \end{aligned} \quad (\text{C.3})$$

The last equality use that $K = 1$ in the above calculations because it is evaluated at an equilibrium. Next, at the equilibrium we have $\lambda = \frac{\bar{b} b_0}{b_0 - \bar{b}}$ and $k_i = \frac{a_i}{a_0} = \frac{b_0(\bar{b} + b_i)}{2\bar{b} b_0 + (b_0 - \bar{b}) b_i}$. Plugging this into equation (C.3) gives

$$\frac{\partial K}{\partial b_0} = \frac{1}{\sum_{j \notin \mathcal{M}} \sigma_j^2 k_j^2} \sum_{j \notin \mathcal{M}} \sigma_j^2 \frac{\bar{b}(\bar{b} + b_0) b_i^2 (b + b_i)}{(2\bar{b} b_0 + (b_0 - \bar{b}) b_i)^3}.$$

This is a sum over positive values since $b_0 - \bar{b} > 0$ if $\lambda > 0$ and we have $\frac{\partial K}{\partial b_0} > 0$ at an equilibrium. Both a_i and a_0 are continuous as long as the second order condition is satisfied and therefore K has to be continuous and there can be at most one solution for λ such that $K = 1$. Define b_k and b_l as the highest and lowest inventory cost for anonymous traders. If $b_0 = b_k$, then $k_i \leq 1$ with $k_i < 1$ if $b_i < b_k$ and $K \geq 1$ with strict inequality if anonymous traders are heterogeneous. The exact opposite holds if $b_0 = b_l$. Therefore we know that there is a $b_0 \in (b_l, b_k)$ such that $K = 1$ if traders are heterogeneous. If traders are homogeneous, $b_0 = b_i$. As a result we can write λ as a convex combination of the λ_i that the anonymous traders would obtain if they revealed their order flow.

The price impact is decreasing in b_i . Hence, the lowest price impact is given by

$$\lambda_i \Big|_{b_i \rightarrow \infty} = \bar{b}.$$

This also puts a bound on λ since it is a convex combination of elements that are all larger than \bar{b} .

C.2.2 Proof of Proposition 8

For brevity, I will do this proof with strictly heterogeneous traders, i.e. $b_i \neq b_j$ for $i \neq j$. This is without loss of generality, and if there are multiple traders with the same parameter b_i , then they would either all strictly prefer the same choice, or be indifferent. As a result, they would be indifferent between all making their choice and letting one of them choose for all traders in the group with the same b_i .

This proof uses part (iii) of Lemma 8 in both parts. In an equilibrium where the price impact for a trader who reveals his or her identity satisfies $\lambda \in [\lambda_j, \lambda_{j+1})$, traders with inventory cost $b_i < b_{j+1}$ strictly prefer to stay anonymous because $\lambda_i \geq \lambda_{j+1}$ if they reveal their identities. Trader j weakly prefers to reveal his or her identity and strictly prefers to do that if $\lambda > \lambda_j$. The opposite is the case for traders with $b_i > b_j$. As a result, any equilibrium must be a threshold equilibrium where traders with higher inventory costs than some threshold reveal their identity. Trader j is indifferent if $\lambda = \lambda_j$.

To show uniqueness of b_j , denote the price impact for traders who trade anonymously as λ^i if every trader $j \geq i$ trade anonymously. Trader i will only trade anonymously if $\lambda^i \leq \lambda_i$. From Lemma 8 we also have $\lambda^i \in [\lambda_i, \lambda^{i+1}]$. Combining the two gives $\lambda^i \geq \lambda^{i+1}$ for $i \geq j$. For $i < j$ we have that $\lambda^i \in [\lambda_i, \lambda^{i+1}]$, which gives $\lambda^i > \lambda_i$. As λ_i is increasing in i , λ^i is decreasing in i for $i \geq j$, and $\lambda^i > \lambda_i$ for $i < j$ there is a unique threshold.

If there are traders who are indifferent between revealing their identity or not, there are multiple equilibria, where all traders will have the same price impacts and behavior regardless of the indifferent traders' choices.

When $\sigma_n^2 = 0$, then the trader with the highest inventory cost will always prefer to reveal his or her identity regardless of what other traders do, and strictly prefer to do so if at least one other trader stays anonymous. As a result, there can not be any equilibrium where he or she stays anonymous except for one where all other traders reveal their type. The trader with the lowest inventory cost always strictly prefers to stay anonymous if there is some probability that other traders do the same. Hence, there can not be any equilibria where the trader with the highest inventory cost stays anonymous. The argument follows for the one with the second-highest inventory cost and so on.

If $b_i = b$ for all b and $\sigma_n^2 > 0$, then $\lambda_i > \lambda$ for all choices of traders to stay anonymous or not, and all traders strictly prefer to stay anonymous.

C.2.3 Proof of Proposition 9

The impatient traders always submit orders $Y_{i1} = -X_i$ regardless of their price impact, which is also the efficient orders in the first round because it minimizes the total inventory costs. Patient traders submit orders $Y_{i1} = -\frac{\bar{b}+b_i}{b_i+2\lambda_i}X_i$. From Lemma 8 we have that $\lambda_i \geq \bar{b}$ and $\lambda \geq \bar{b}$. The inequalities are strict when traders have $b_i < \infty$. As a result, $\frac{\bar{b}+b_i}{b_i+2\lambda_i} < 1$ and the market orders from patient traders are too small to be efficient (i.e. $Y_{i1} = -X_i$). $\frac{\bar{b}+b_i}{b_i+2\lambda_i}$ is decreasing in λ_i and total welfare is higher if patient traders obtain a lower price impact.

From Lemma 8 part (iii), we know that patient traders obtain a strictly lower price impact if they pool with the impatient traders and vice versa for impatient traders. From part (ii) of the Lemma, we know that this means that patient (impatient) traders have higher (lower) welfare in an anonymous market.

C.2.4 Lemma 9

Suppose that the price impact for trader i if he trades anonymously will be $\tilde{\lambda}_i$ and that he is indifferent between trading anonymously and revealing his order flow. This gives the following equation:

$$E[E[U_i|X_i]] = E\left[\frac{(\bar{b}^2 + 2\bar{b}b_i - 2b_i\tilde{\lambda}_i)X_i^2}{2(b_i + 2\tilde{\lambda}_i)} + A_i\right] = E\left[\frac{(\bar{b}^2 + 2\bar{b}b_i - 2b_i\lambda_i)X_i^2}{2(b_i + 2\lambda_i)} + A_i - c_i\right]$$

with $\lambda_i = \frac{\bar{b}b_i}{b_i - \bar{b}}$. This simplifies to

$$\frac{(\bar{b}^2 + 2\bar{b}b_i - 2b_i\tilde{\lambda}_i)\sigma_i^2}{2(b_i + 2\tilde{\lambda}_i)} = \frac{(\bar{b}^2 + 2\bar{b}b_i - 2b_i\lambda_i)\sigma_i^2}{2(b_i + 2\lambda_i)} - c_i.$$

The unique solution to this equation is $\tilde{\lambda}_i = \frac{b_i(b_i + \bar{b}(\bar{b} + b_i)\frac{\sigma_i^2}{c_i})}{(b_i^2 - \bar{b}^2)\frac{\sigma_i^2}{c_i} - 2b_i}$.

C.2.5 Proof of Proposition 10

Rank the traders such that $\tilde{\lambda}_1 \leq \tilde{\lambda}_2 \leq \dots$. Consider a potential equilibrium where some traders reveal their order flow. Then there is either (1) an $i \in \{1, \dots, N\}$ such that every trader $j \leq i$ prefers to reveal his order flow and every trader $j > i$ prefers to trade anonymously or (2) all traders prefer to trade anonymously. The equilibrium is not necessarily unique as the decisions to rationally reveal order flow are strategic complements. That is, trader i will only reveal his order flow if it reduces his price impact, and thus by Lemma 8 part (iii) increases the price impact of trading anonymously. This choice increases the incentives for trader $i + 1$ to reveal his order flow.

C.2.6 Proof of Proposition 11

Frankel et al. (2003) shows that there is a unique threshold equilibrium given the following set of assumptions:

- (A1) Strategic complementarities. Higher actions from other players weakly increase the incentives to choose higher actions.
- (A2) Unique equilibrium regions.
- (A3) State monotonicity. A higher state increases the incentives for higher actions.
- (A4) Payoff continuity in all parameters.
- (A5) Bounded derivatives (irrelevant for finite action games).

Given the assumptions, there is a unique increasing pure strategy that survives the deletion of strictly dominated strategies (Theorem 1 in their paper).

Players only reveal their order flow if it reduces their price impact. The distribution of the price impact for anonymous traders is weakly higher when more traders can reveal their order flow and (A1) holds. No traders have an incentive to pay a cost c_i if V is sufficiently close to 0 because the potential for increasing their welfare from a lower price impact goes towards 0 while the cost is positive, and there is a lower dominance region for all traders. There is an upper dominance region for all traders except the one(s) with the lowest holding cost. If V is sufficiently large, then c_i is tiny relative to the benefit of a small reduction in the price impact, and we are almost at the equilibrium in Proposition 8 when $\sigma_n^2 = 0$. Therefore there are two dominance regions and (A2) holds. A higher state increases the incentives to reduce the price impact since the expected trading needs are larger, whereas the cost is the same, and (A3) holds. The expected payoff is a continuous function of the variance of shocks and (A4) holds. (A5) is not necessary in a game with finite actions. Hence, all assumptions are satisfied, and there is a unique threshold equilibrium.

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1. *Time-varying liquidity with strategic traders*
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SEMINARS AND CONFERENCE PRESENTATIONS

2020

BI (1), SFI research days (4)

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Teaching Assistant, Mathematics for Economists, Bachelor, 2012-2014
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